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CBCS SCHEME

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21MAT31

Third Semester B.E. Degree Examination, June/July 2023
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

VTU-04-10-2023 09:05:00am - AY - AY - AY
VTU-04-10-2023 09:06:46am - AY - AY - AY

Module-1

1 a. Find the Laplace transform
$$2^t + \frac{\cos 2t + \cos 3t}{t}$$ (06 Marks)

b. Find the Laplace transform of the triangular wave of period 2C given by
$$f(t) = \begin{cases} t & 0 < t < c \\ 2c - t & c < t < 2c \end{cases}$$ (07 Marks)

c. Using convolution theorem find the inverse Laplace transform of
$$\frac{s}{(s^2 + a^2)^2}$$ (07 Marks)

OR

2 a. Express the function f(t) in terms of unit step function and hence find the Laplace transform of
$$f(t) = \begin{cases} \sin t & 0 < t < \pi \\ \sin 2t & \pi < t < 2\pi \\ \sin 3t & t \geq 2\pi \end{cases}$$ (06 Marks)

b. Find the inverse laplace transform $\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}$ (07 Marks)

c. Solve the using Laplace transform method
$$y''(t) + 4y'(t) + 4y = e^{-t} \quad y(0) = 0 \quad y'(0) = 0$$ (07 Marks)

Module-2

3 a. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that
$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots = \frac{\pi}{4}$$ (06 Marks)

b. Obtain the half range cosine series for the function $f(x) = 2x - 1$ in $0 < x < 1$ (07 Marks)

c. Obtain the Fourier series of y upto the first harmonic for the following values:

x°	45	90	135	180	225	270	315	360
y	4.0	3.8	2.4	2.0	-1.5	0	2.6	3.4

(07 Marks)

OR

4 a. Obtain the Fourier series of $f(x) = x\cos x$ in the interval $-\pi \le x \le \pi$. **(06 Marks)**

b. Obtain the sine half range Fourier series for the function,

$$f(x) = \begin{cases} \dfrac{2Kx}{\ell} & \text{in } 0 \le x \le \dfrac{\ell}{2} \\ \dfrac{2K}{\ell}(\ell - x) & \text{in } \dfrac{\ell}{2} \le x \le \ell \end{cases}.$$

(07 Marks)

c. Obtain the constant term and the first three coefficients in the Fourier cosine series of y in the following data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(07 Marks)

Module-3

5 a. Find the complex Fourier transform of the function,

$$f(x) = \begin{cases} a^2 - x^2 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}.$$

Hence evaluate $\displaystyle\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right) ds = \frac{\pi}{2}$. **(06 Marks)**

b. Find the Fourier sine transform of e^{-ax}. **(07 Marks)**

c. Find the z-transform of $\cos n\theta$ and $\sin n\theta$. **(07 Marks)**

OR

6 a. Find the Fourier cosine transform of the function, $f(x) = \begin{cases} 4x & 0 < x < 1 \\ 4 - x & 1 < x < 4 \\ 0 & x > 4 \end{cases}$. **(06 Marks)**

b. Find the inverse z-transform of $\dfrac{2z^2 + 3z}{(z+2)(z-4)}$. **(07 Marks)**

c. Solve by using z-transform $y_{n+2} - 4y_n = 0$ given that $y_0 = 0$ and $y_1 = 2$. **(07 Marks)**

Module-4

7 a. Classify the following partial differential equation

i) $\dfrac{\partial^2 u}{\partial x^2} + 4\dfrac{\partial^2 u}{\partial x \partial y} + 4\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial y}{\partial x} + 2\dfrac{\partial u}{\partial y} = 0$

ii) $x^2 \dfrac{\partial^2 u}{\partial x^2} + (1 - y^2)\dfrac{\partial^2 u}{\partial y^2} = 0 \quad -\infty < x < \infty, -1 < y < 1$

iii) $(1 + x^2)\dfrac{\partial^2 u}{\partial x^2} + (5 + 2x^2)\dfrac{\partial^2 u}{\partial x \partial t} + (4 + x^2)\dfrac{\partial^2 u}{\partial t^2} = 0$

iv) $(x + 1)\dfrac{\partial^2 u}{\partial x^2} - 2(x + 2)\dfrac{\partial^2 u}{\partial x \partial y} + (x + 3)\dfrac{\partial^2 u}{\partial y^2} = 0$ **(10 Marks)**

b. Find the values of u(x, t) satisfying the parabolic equation $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ and its boundary conditions u(0, t) = 0 = u (4,t) and u(x, 0) = x(4 – x) by taking h = 1 find the value up to t = 5. **(10 Marks)**

OR

8 a. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5, t \geq 0$ given that u(x, 0) = 20 u(0, t) = 0 u(5,t) = 100 compute U for the time step h = 1 by crank Nicholson method. **(10 Marks)**

b. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the condition u(0, t) = 0 u(4, t) = 0 $u_t(x, 0) = 0$ and u(x, 0) = x(4 – x) by taking h = 1, K = 0.5 up to four steps. **(10 Marks)**

Module-5

9 a. Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, y(0)) = 1, y′(0) = 0 evaluate y(0.1) using Runge-Kutta method of order 4. **(06 Marks)**

b. Derive the Euler's equation of the form $\frac{\partial t}{\partial y} - \frac{d}{dx}\left(\frac{\partial t}{\partial y1}\right) = 0$. **(07 Marks)**

c. Find the extremal of the functional $I = \int_0^{\pi/2}(y^2 - y'^2 - 2y\sin x)dx$ under the conditions y(0) = y(π/2) = 0. **(07 Marks)**

OR

10 a. Apply Milne's predictor corrector method to solve $\frac{d^2y}{d^2x} = 1 - 2y\frac{dy}{dx}$ at 0.8 given that

y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762,
y′(0) = 0, y′(0.2) = 0.1996, y′(0.4) = 0.3937, y′(0.6) = 0.5689. **(06 Marks)**

b. Show that the geodesics on a plane are straight line. **(07 Marks)**

c. Which curve the functional $\int_0^{\pi/2}(y'^2 - y^2 + 2xy)dx$, y(0) = 0, y(π/2) = 0 be extremized. **(07 Marks)**

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