60856

Reg. No.

I Semester M.Sc. Degree Examination, June/July - 2022

MATHEMATICS

Mathematical Analysis

(CBCS Scheme 2018-19 and 2020 Onwards Y2K17 Repeaters)

Paper : M- 107 SC

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer any **five** full questions.
2. All questions Carry **equal** marks.

1. a. Prove that $f: X \to Y$ be continuous function, where X and Y are metric space, iff $f^{-1}(V)$ is open in X, where V is open in Y.

b. Examine the function,

$$f(x)=\begin{cases}\dfrac{\cos x}{\pi/2-x}, & x\neq \pi/2\\ 1, & x=\pi/2\end{cases}$$

for continuity at $x=\pi/2$.

c. Show that every differentiable function is continuous. But converse is need not be, justify with an example. **(6+4+4)**

2. a. Let $f: X \to Y$ be continuous mapping, when X is compact metric space, then show that f is uniformly continuous on X.

b. Show that continuous image of a connected set is connected.

c. Let f be a real uniformly continuous function on the bounded set A in $\mathbb{R}'$. **(6+4+4)**

3. a. State and prove mean value theorems.

b. Verify Lagrange's mean value theorem for the function $f(x)=x(x-1)(x-2)$ in [0,1/2]. **(9+5)**

[P.T.O.

4. a. Show that a sequence $\{a_n\}$ is bounded if and only if there exists a positive real number M such that $|a_n| \leq M \ \forall n \in N$.

b. Show that the sequence $\{a_n\}$ defined by $a_n = \left(1+\frac{1}{n}\right)^n$ is covergent. **(7+7)**

5. a. Define upper limit and lower limit. Let $\{a_n\}$ is a bounded sequence, then show that the followings:

i. $\overline{\lim_{n\to\infty}}(-a_n) = -\underline{\lim_{n\to\infty}}\, a_n$.

ii. $\underline{\lim_{n\to\infty}}(-a_n) = -\overline{\lim_{n\to\infty}}\, a_n$.

iii. $\overline{\lim_{n\to\infty}}(\lambda a_n) = \lambda \overline{\lim_{n\to\infty}}\, a_n$, $\lambda > 0$.

iv. $\underline{\lim_{n\to\infty}}(\lambda a_n) = \lambda \underline{\lim_{n\to\infty}}\, a_n$, $\lambda > 0$.

b. Show that every cauchy sequence is bounded. **(10+4)**

6. a. Examine the convergence of the series $\frac{3}{5}+\frac{4}{5^2}+\frac{3}{5^3}+\frac{4}{5^4}+, \ldots\ldots$

b. State and prove D'Alembert's ratio test.

c. Let P>0, then show that $\lim_{n\to\infty} \sqrt[n]{P} = 1$. **(4+6+4)**

7. a. Show that e is irrational.

b. Let $\sum a_n$ is a series of complex numbers which converges absolutely, then show that rearrangement of $\sum a_n$ converges, and they all converges to the same sum.

c. State and prove Cesaro's limit theorem. **(4+5+5)**

8. a. State and prove Bolzano weierstrass theorem.

b. State and prove Heine - Borel theorem.

c. Prove that $\mathbb{R}'$ is not compact. **(4+6+4)**
