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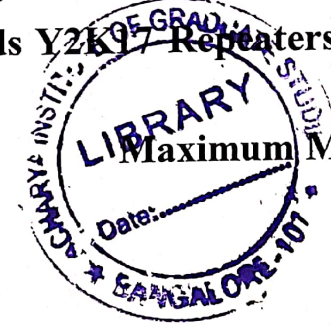
I Semester M.Sc. Degree Examination, June/July - 2022

MATHEMATICS

Mathematical Analysis

(CBCS Scheme 2018-19 and 2020 Onwards Y2K17 Repeaters)

Paper : M- 107 SC



Time : 3 Hours

Instructions to Candidates:

1. Answer any **five** full questions.
2. All questions Carry **equal** marks.

1. a. Prove that $f : X \rightarrow Y$ be continuous function, where X and Y are metric space, iff $f^{-1}(V)$ is open in X , where V is open in Y .

- b. Examine the function,

$$f(x) = \begin{cases} \frac{\cos x}{\pi/2 - x}, & x \neq \pi/2 \\ 1, & x = \pi/2 \end{cases}$$

for continuity at $x = \pi/2$.

- c. Show that every differentiable function is continuous. But converse is need not be, justify with an example. (6+4+4)

2. a. Let $f : X \rightarrow Y$ be continuous mapping, when X is compact metric space, then show that f is uniformly continuous on X .

- b. Show that continuous image of a connected set is connected.

- c. Let f be a real uniformly continuous function on the bounded set A in \mathbb{R}' . (6+4+4)

3. a. State and prove mean value theorems.

- b. Verify Lagrange's mean value theorem for the function $f(x) = x(x-1)(x-2)$ in $[0, 1/2]$. (9+5)

[P.T.O.]





4. a. Show that a sequence $\{a_n\}$ is bounded if and only if there exists a positive real number M such that $|a_n| \leq M \forall n \in \mathbb{N}$.

b. Show that the sequence $\{a_n\}$ defined by $a_n = \left(1 + \frac{1}{n}\right)^n$ is convergent. (7+7)

5. a. Define upper limit and lower limit. Let $\{a_n\}$ is a bounded sequence, then show that the followings:

i. $\overline{\lim}_{n \rightarrow \infty} (-a_n) = -\lim_{n \rightarrow \infty} a_n$.

ii. $\lim_{n \rightarrow \infty} (-a_n) = -\overline{\lim}_{n \rightarrow \infty} a_n$.

iii. $\overline{\lim}_{n \rightarrow \infty} (\lambda a_n) = \lambda \overline{\lim}_{n \rightarrow \infty} a_n, \lambda > 0$.

iv. $\lim_{n \rightarrow \infty} (\lambda a_n) = \lambda \lim_{n \rightarrow \infty} a_n, \lambda > 0$.

b. Show that every cauchy sequence is bounded. (10+4)

6. a. Examine the convergence of the series $\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} + \dots$

b. State and prove D'Alembert's ratio test.

c. Let $P > 0$, then show that $\lim_{n \rightarrow \infty} \sqrt[n]{P} = 1$. (4+6+4)

7. a. Show that e is irrational.

b. Let $\sum a_n$ is a series of complex numbers which converges absolutely, then show that rearrangement of $\sum a_n$ converges, and they all converges to the same sum.

c. State and prove Cesaro's limit theorem. (4+5+5)

8. a. State and prove Bolzano weierstrass theorem.

b. State and prove Heine - Borel theorem.

c. Prove that \mathbb{R}' is not compact. (4+6+4)

