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I Semester M.Sc. Degree Examination, June/July - 2022

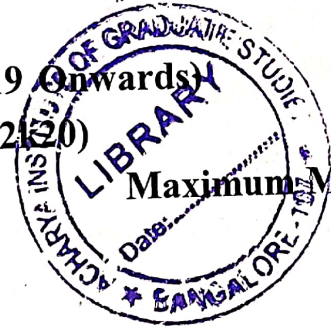
MATHEMATICS

Ordinary Differential Equations

Paper : M- 104 T

(CBCS Scheme Repeaters 2018-19 Onwards)

(Common to Y2k17 and Y2k20)



Maximum Marks : 70

Time : 3 Hours

Instructions to Candidates:

1. Answer any **Five** questions.
2. All questions carry **equal** marks.

1. a. Define Wronskian of $y_1(x), y_2(x), y_3(x), \dots, y_n(x)$. Show that the solutions of $y''''(x) - y''''(x) - y'(x) + y(x) = 0$ are linearly independent.
b. Show that the solutions of an n^{th} order differential equation form a fundamental set. (9+5)
2. a. State and prove the Sturm's separation theorem.
b. Show that $y'' + \left(\frac{1}{4x^2} + \frac{1}{(x \log x)^2} \right) y = 0$, $k > 0$ is some constant, is oscillatory if $k > 1/4$ and non - oscillatory if $k < 1/4$. (7+7)
3. a. Define the existence and uniqueness theorem. Use the same theorem to show $y'(x) = y^2(x), y(1) = -1$ where $|x-1| \leq a, |y+1| \leq b$, for some constants a and b , has a unique solution in the range $0.75 \leq x \leq 1.25$.
b. Find all the eigen values and eigen functions of $\frac{d}{dx}(xy') + \frac{\lambda}{x}y = 0, y'(0) = 0 = y'(e^{2\pi})$. (7+7)
4. a. Establish the Green's function for the eigenvalue problem.
b. Solve $y'' + 0.25y = \sin(2\pi x)$ with $y(0) = 0 = y(\pi)$ by constructing the Green's function. (7+7)

[P.T.O.]





5. a. Determine the ordinary, regular and irregular singular point of $(1-x^2)y''(x) - xy'(x) / x^2 y(x) = 0$. Also discuss a singular point at ∞ .
- b. Find the power series solution of $(x^2 - 1)y'' + 3xy' + xy = 0$ with $y(0) = 4$, $y'(0) = 6$.
(7+7)
6. a. Show that Hermite polynomials are orthogonal over $(-\infty, \infty)$.
- b. Show that Chebyshev polynomials are orthogonal over $[-1, 1]$.
(9+5)
7. a. Express an n^{th} differential equation as a system of first order differential equations.
- b. Find the fundamental matrix and general solution of $\bar{X}'(t) = A\bar{X}(t)$ where $A = [5, 4; 1, 2]$.
(5+9)
8. a. Locate the critical point and find its nature for the system $\frac{dx}{dt} = x + y$, $\frac{dy}{dt} = 3x - y$.
Also find the equation of phasepath.
- b. Determine the stability of the critical point $(0, 0)$ of $\frac{dx}{dt} = -x^3 - 3xy^4$,
 $\frac{dy}{dt} = x^2y - 2y^3 - y^5$ using Liapunov function.
(7+7)

