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Reg. No. [ ]

I Semester M.Sc. Degree Examination, June/July - 2022

MATHEMATICS

Ordinary Differential Equations

Paper : M- 104 T

(CBCS Scheme Repeaters 2018-19 Onwards)

(Common to Y2k17 and Y2k20)

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer any **Five** questions.
2. All questions carry **equal** marks.

1. a. Define Wronskian of $y_1(x), y_2(x), y_3(x), ..., y_n(x)$. Show that the solutions of $y''''(x) - y'''(x) - y'(x) + y(x) = 0$ are linearly independent.

b. Show that the solutions of an n^{th} order differential equation form a fundamental set. **(9+5)**

2. a. State and prove the Sturm's separation theorem.

b. Show that $y'' + \left(\frac{1}{4x^2} + \frac{1}{(x \log x)^2}\right) y = 0$, k > 0 is some constant, is oscillatory if k > 1/4 and non - oscillatory if k < 1/4. **(7+7)**

3. a. Define the existence and uniqueness theorem. Use the same theorem to show $y'(x) = y^2(x), y(1) = -1$ where $|x-1| \le a, |y+1| \le b$, for some constants a and b, has a unique solution in the range $0.75 \le x \le 1.25$.

b. Find all the eigen values and eigen functions of $\frac{d}{dx}(xy') + \frac{\lambda}{x} y = 0$, $y'(0) = 0 = y'(e^{2\pi})$. **(7+7)**

4. a. Establish the Green's function for the eigenvalue problem.

b. Solve $y'' + 0.25\,y = \sin(2\pi)$ with $y(0) = 0 = y(\pi)$ by constructing the Green's function. **(7+7)**

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5\. a. Determine the ordinary, regular and irregular singular point of $(1-x^2)y''(x)-xy'(x)/x^2y(x)=0$. Also discuss a singular point at ∞.

b. Find the power series solution of $(x^2-1)y''+3xy'+xy=0$ with $y(0)=4,\ y'(0)=6$.

(7+7)

6\. a. Show that Hermite polynomials are orthogonal over $(-\infty,\infty)$.

b. Show that Chebyshev polynomials are orthogonal over [-1,1]. **(9+5)**

7\. a. Express an n^{th} differential equation as a system of first order differential equations.

b. Find the fundamental matrix and general solution of $\vec{X}'(t)=A\vec{X}(t)$ where A=[5,4; 1,2]. **(5+9)**

8\. a. Locate the critical point and find its nature for the system $\frac{dx}{dt}=x+y,\ \frac{dy}{dt}=3x-y$. Also find the equation of phasepath.

b. Determine the stability of the critical point (0,0) of $\frac{dx}{dt}=-x^3-3xy^4$, $\frac{dy}{dt}=x^2y-2y^3-y^5$ using Liapunov function. **(7+7)**