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Reg. No.

II Semester M.Sc. Degree Examination, November - 2022

MATHEMATICS

Elementary Number Theory

(CBCS Scheme Y2k17/2020)

Paper : M207SC

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates: **1) Answer any FIVE questions**

2) All questions have equal marks.

1. a) Find gcd (1485,1745) using division algorithm. Hence find *l*cm (1485,1745). **(7+7)**

   b) State and prove Bezout's identity which expresses gcd (a,b) as a linear combination of a and b.

2. a) State and Prove fundamental theorem of arithmetic. **(7+7)**

   b) If the square root of a positive integer m is rational, then show that m is a perfect Square.

3. a) If $ca \equiv cb$ mod (n), then prove that $a \equiv b \bmod \left[\frac{n}{d}\right]$, where d=gcd (c,n). **(4+5+5)**

   b) Let $ax \equiv b$ (mod n) and gcd (a,n) = d then prove that the congruence has exactly d-solutions, if d/b.

   c) Find all integers 'n' for which $n^{13} \equiv n$ (mod1365).

4. a) State and prove the Fermat's little theorem. **(5+4+5)**

   b) State and prove the Chinese Remainder theorem for simultaneous Congruences.

   c) Solve the simultaneous congruence:

   $7x \equiv 3 \bmod (12)$

   $10x \equiv 6 \bmod (14)$

[P.T.O

5. a) Let p be an odd prime and gcd (a,p) = 1. Then show that a is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1$ (mod p). **(7+7)**

   b) Apply Gauss' lemma to determine whether 3 and 5 are quadratic residues mod (29). Does 10 belong to Q29?

6. a) Is 83 a quadratic residue mod (103)? Substantiate your answer. **(5+5+4)**

   b) Show that there are infinitely many primes of the form 8k-1.

   c) Show that 3 is a quadratic residue of mod 23.

7. a) Show that an odd prime p is expressible as a sum of two squares if and only if p ≡ 1 (mod 4) **(5+4+5)**

   b) Prove that no prime p of the form 4k+3 is a sum of 2-squares.

   c) Express the following integers as sum of four squares : 247, 308.

8. a) Write down the Pythagorean triple (3,4,C) that can be obtained from a right-angled triangle whose two sides are 3 and 4. Explain a simple method of further obtaining Pythagorean triples from (3,4,C). With suitable explanation identify any one primitive Pythagorean triple among the obtained Pythagorean triples. **(7+7)**

   b) Show that there are no positive integer solutions x, y and z of $x^4 + y^4 = z^4$.
