60871

Reg. No. [ ]

III Semester M.Sc. Degree Examination, April/May - 2022

MATHEMATIC

Differential Geometry

(CBCS Scheme Repeater Y2K17)

Paper - M301 T

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates :

Answer any Five questions. All questions carry Equal marks.

1. a) Define :

i) a tangent vector to E^3

ii) a vector field on E^3

iii) Directional derivative in E^3

Further, if $v = (2,-1,3)$ and $p = (2,0,-3)$. Then compute the directional derivative $v_p[f]$ for the functions :

i) $f = y^2z$ ii) $f = x^7$

iii) $f = e^x \cos y$

b) Define a curve in E^3. Let α be a curve in E^3 and let f be a differentiable function on E^3. Then show that $\alpha^1(t)[f] = \frac{d(f(\alpha))}{dt}(t)$

c) Let $v = (1,2-3)$ and $p = (0,-2,1)$. Evaluate the following 1-forms on the tangent vector up.

i) $y^2\,dx$ ii) $zdy - ydz$ iii) $(z^2-1)\,dx-dy+x^2dz$

(5+4+5)

2. a) Let ϕ and ψ be two 1-forms. Then prove the Leibnizian formula $: d(\phi\wedge\psi) = d\phi\wedge\psi - \phi\wedge d\psi$.

b) Let F = $(f_1, f_2, \ldots\ldots f_m)$ be a mapping from E^n to E^m. If v is a tangent vector to E^n at p, then prove that $F_*(v)(v[f_1], v[f_2] \ldots\ldots v[f_m])$ at $F(p)$.

(c) For any three 1-forms $\phi_i = \sum_j f_i\,dx_j\,(1 \le i \le 3)$, Prove

$$\phi_1\wedge\phi_2\wedge\phi_3 = \begin{vmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{vmatrix} dx_1\, dx_2\, dx_3$$

(6+4+4)

[P.T.O.

3. a) If α is a regular curve in E^3, then prove that there exists a reparametrization β of α such that β has unit speed. Further, show that a helix given by

$$\beta(s)=\left(a\cos\frac{s}{c}, a\sin\frac{s}{c}, b\frac{s}{c}\right),$$ where $c^2=\sqrt{a^2+b^2}$ & $a>0$, has a unit speed.

b) If α is a regular curve with curvature function $k>0$ and torsion function C, then show that α is a cylindrical helix if and only if the ratio c/k is constant.

c) If α is a regular curve in E^3, then show that

$$T=\alpha'/\|\alpha'\|,\ N=B\times T,\ B=\alpha'\times\alpha''/\|\alpha'\times\alpha''\|.$$

$$K=\|\alpha'\times\alpha''\|/\|\alpha'\|^3,\ C=(\alpha'\times\alpha'').\alpha'''/\|\alpha'\times\alpha''\|^2$$

(5+4+5)

4. a) Consider the tangent vector $v=(1,-1,2)$ at a point $p=(1,3,-1)$. computer $\nabla_v W_1$ where

i) $W=x^2u_1+yu_2$

ii) $W=xu_1+x^2u_2-z^2u_3$.

b) If F is an isometry of E^3 such that F (o) = O, then show that F is an orthogonal transformation.

c) If $C=\begin{bmatrix} -2/3 & 2/3 & -1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}$ and $\begin{cases} p=(3,1,-6) \\ q=(1,0,3). \end{cases}$ show that C is orthogonal then compute c(p) and c(q). Further, check that c(p), c(q) = p.q.

(4+5+5)

5. a) Let f be a real valued differentiable function on a non-empty open set D of E^2, then prove that the function $X:D\rightarrow E^3$ defined by $X(u,v)=(u,v,f(u,v))$ is a proper patch in E^3.

b) Let X : $E^2\rightarrow E^3$ defined by $X(u,v)=(u+v,u-v,x)$ show that X is a proper patch and that image of X is a surface M such that $z=\frac{x^2-y^2}{4}$, where $(x,y,z)Ex$ (D) and $(u,v)\in D\leq E^2$.

c) Show that every cylinder in E^3 is a surface in E^3.

(5+5+4)

6. a) Let X be a mapping from a non-empty open set D of E^2 to E^3. The show that X is regular if and only if the x, v - parameter partial derivatives X_x(d) and X_v (d) are linearly independent for all dED.

b) Let P be a point of a surface in E^3 and X be a patch in M such that $X(v_o, v_o) = p$. prove a tangent vector v at a point P is tangent to M if and only if v can be written as a linear combination of $X_u(u_o, v_o)$ and $X_v(u_o, v_o)$. **(7+7)**

7. a) Let ϕ be a 1-form an a surface M. If X and Y are the patches in M defined on D and E respectively, then prove that $dx\phi = dy\phi$ on the overlap of x(D) and Y(E).

b) Let $F : M \to N$ be a mapping of surfaces and let ξ *and* η be forms on N. Then prove the following :

i) $F^*(\xi \wedge \eta) = F^*\xi \wedge F^*\eta$

ii) $F^*(d\xi) = d(F^*\xi)$ **(7+7)**

8. a) Define a shape operator, for each point of a surface M in E^3, Further, Show that the shape operator is a linear operator $Sp : Tp(m) \to Tp(m)$ on the tangent plane of M at a Point P.

b) With usual notation prove:

a) $K(x) = \dfrac{Ln - m^2}{EG - F^2}$

b) $H(x) = \dfrac{Gl + En - 2Fm}{2(EG - F^2)}$

c) Compute Gaussian Curvature K and mean Curvature H of helicoid $X(u, v) = (u\cos v,\ ufinv, bv), b \neq o$. **(5+5+4)**