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III Semester M.Sc. Degree Examination, April/May - 2022

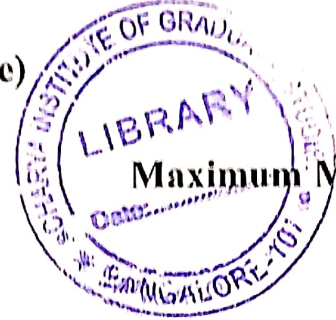
MATHEMATICS

Functional Analysis

Paper : M-303 T

(CBCS Y2K17 Scheme)

(Rep)



Time : 3 Hours

Instructions to Candidates:

- i. Answer any FIVE full questions.
- ii. All questions carry Equal marks.

1. a. Define a normed linear space. Show that in a normed linear space following hold.
 - i. Norm is continuous.
 - ii. Addition and scalar multiplication is jointly continuous.
- b. If M is a closed linear subspace of a normed linear space N , then prove that N/M is a normed linear space. Further, If N is a Banach space, then show that N/M is also Banach space. (6+8)
2. a. Prove that the linear space $B(N, N')$ of all continuous linear transformations of N into N' is a Normed linear space, where $\|T\| = \text{Sup}\{\|Tx\| : \|x\| \leq 1\}$, $\forall T \in B(N, N')$ further show that $B(N, N')$ is complete where N' is complete.
- b. Show that there is an isometric isomorphism of a normed linear space into its second dual. (8+6)
3. a. If N is a normed linear space and $x_0 \in N$ with $x_0 \neq 0$, then show that there exists $f_0 \in N^*$ such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.
- b. State the open mapping theorem prove the closed graph theorem. (6+8)
4. a. State and prove uniform boundedness theorem.
- b. Show that the mapping $T \rightarrow T^*$ is an isometric isomorphism of $B(N)$ into $B(N^*)$ which reverse the product and preserves the identity transformation, where T is an operator on N and T^* an operator on N^* . (7+7)

[P.T.O.]





5. a. Define a Hilbert space and show that every inner product space is a normed linear space.
- b. Show that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm. (7+7)
6. a. Define orthonormal complement of a set S in a Hilbert space H . Prove the following:
- $S \cap S^\perp \leq \{0\}$.
 - $S_1 \leq S_2 \Rightarrow S_2^\perp \leq S_1^\perp$.
 - S^\perp is a closed sub space of H .
 - $S \subseteq S^{\perp\perp}$.
- b. If $\{e_1, e_2, \dots, e_n\}$ is a finite orthonormal set in a Hilbert space H and $x \in H$, then prove that
- $\sum_{i=1}^n |\langle x, e_i \rangle|^2 \leq \|x\|^2$.
 - $x - \sum_{i=1}^n \langle x, e_i \rangle e_i \perp e_j \forall j$. (8+6)
7. a. Define the conjugate space H^* of a Hilbert space H . For every functional f on H , prove that there is a unique $y \in H$ such that $f(x) = \langle x, y \rangle \forall x \in H$.
- b. Define self - adjoint operators on H , prove that the set of all self - adjoint operators in $B(H)$ form a closed real linear space of H and contain I .
- c. If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other then prove that $N_1 + N_2$ and N_1, N_2 are normal. (6+4+4)
8. a. Define unitary operator on H . Prove that T is unitary iff it is an isometric isomorphism of H onto itself.
- b. State and prove spectral theorem. (6+8)

