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Reg. No. [| | | | | | | |]

III Semester M.Sc. Degree Examination, April/May - 2022

MATHEMATICS

Numerical Analysis - II

Paper : M-305 T

(CBCS Scheme Y2K17)

(Rep)

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. *Answer any FIVE questions.*
2. *All questions carry Equal marks.*

1. a. Establish Taylor's series method for $y' = f(x, y)$, $x > x_0$ subjected to the condition $y(x_0) = y_0$, and solve the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$; $y(0) = 0$, $y'(0) = 1$, by Taylor's series method so that the truncation error is not greater than $\frac{1}{2} \times 10^{-4}$ for $x \leq 0.2$. **(10)**

   b. Find the approximate solution by picard's method for the differential equation $\frac{dy}{dx} = x^2 - y, y(0) = 1$ which is correct within an accuracy of 10^{-3} for $0 \leq x \leq 0.2$. **(4)**

2. a. Derive Euler's modified formula to solve the differential equation $y' = f(x, y)$ at $y(x_0) = y_0$. Find also discuss about its error. **(7)**

   b. Using Euler's method, compute y_1 and y_2 taking $h = 0.1$ from the following differential equation $\frac{dy}{dx} = 1 + xy^2$, $y(0) = 1$. Also, compute the error in both. **(7)**

3. Derive the Adam - Bashforth and Adam - Moulton's third and fourth order methods for $y' = f(x, y), y(x_0) = y_0$. **(14)**

4. Describe the method of shooting technique for the solution of the higher order differential equation. And hence apply the same technique to solve $y'' + 6xy' + 5y = x^2$ with $y(0) = 1$, $y(1) = 0$. **(14)**

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5. Solve the equations $\frac{\partial u}{\partial t}=-16\frac{\partial^2 u}{\partial x^2}$ subjected to the boundary conditions $u(x,0)=Sin[\pi x]$, $0\le x\le 1$, $u(0,t)=u(1,t)=1$ using

a. Schmidt method.

b. Crank - Nicolson method.

c. Dufort - Frankel method.

Take $\Delta x=0.25$, $\Delta t=\frac{1}{36}$. **(14)**

6. a. A tightly streched string with fixed end points x = 0, and x = 1.0 is at rest in its equilibrium positions. At t = 0, each point of the string is given a velocity 20x(1-x). Find the displacement of the string at x = 0(0.1)1.0 for t = 0(0.1) 1.0 by finite differences using explicit method. Consider the normal form $\frac{d^2u}{dt^2}=\frac{d^2u}{dx^2}$ for vibrating string. **(7)**

b. Use the method of characteristics to find the solution of the non - linear equation $\frac{\partial^2 u}{\partial x^2}-u^2\frac{\partial^2 u}{\partial y^2}=0$ at the first characteristic grid point between x = 0.2 and 0.3, y >0, where u satisfies the condition $u = 5x^2$, and $\frac{\partial u}{\partial y}=2x$ along the initial line y = 0, $0\le x\le 1$. **(7)**

7. a. Solve the heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subjected to the boundary conditions u(x,0) = 0, u(0,t) = 0, u(1,t) = t. Take $\Delta t=\frac{1}{36}$, $\Delta x=0.25$. **(7)**

b. Obtain the solution of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=Sin(\pi x).Sin(\pi y)$, $0\le x,y\le 1$ with u=0 on the square boundary. Take $\Delta x=\Delta y=\frac{1}{3}$. **(7)**

8. Find the solution of $U_t=U_{xx}+U_{yy}$, $0\le x,y\le 1$ with conditions $U(x,y,0)=Sin(\pi x).Sin(\pi y)$, $U_t(x,y,0)=0$, and U = 0 on the boundary. Take $\Delta x=\Delta y=\frac{1}{3}, \Delta t=\frac{1}{9}$. Perform one time integration using first Lees alternating direction implicit method. **(14)**