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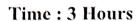
## III Semester M.Sc. Degree Examination, April/May - 2022 MATHEMATICS

## Numerical Analysis - II

Paper: M-305 T

(CBCS Scheme Y2K17)

(Rep)



Instructions to Candidates:

- 1. Answer any FIVE questions.
- 2. All questions carry Equal marks.



- 1. a. Establish Taylor's series method for y' = f(x, y),  $x > x_0$  subjected to the condition  $y(x_0) = y_0$ , and solve the differential equation  $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ ; y(0) = 0, y'(0) = 1, by Taylor's series method so that the truncation error is not greater than  $\frac{1}{2} \times 10^{-4}$  for  $x \le 0.2$ .
  - b. Find the approximate solution by picard's method for the differential equation  $\frac{dy}{dx} = x^2 y, y(0) = 1 \text{ which is correct within an accuracy of } 10^{-3} \text{ for } 0 \le x \le 0.2.$  (4)
- 2. a. Derive Euler's modified formula to solve the differential equation y' = f(x, y) at  $y(x_0) = y_0$ . Find also discuss about its error. (7)
  - b. Using Euler's method, compute  $y_1$  and  $y_2$  taking h = 0.1 from the following differential equation  $\frac{dy}{dx} = 1 + xy^2$ , y(0) = 1. Also, compute the error in both. (7)
- 3. Derive the Adam Bashforth and Adam Moulton's third and fourth order methods for  $y' = f(x, y), y(x_0) = y_0$ . (14)
- 4. Describe the method of shooting technique for the solution of the higher order differential equation. And hence apply the same technique to solve  $y'' + 6xy' + 5y = x^2$  with y(0) = 1, y(1) = 0. (14)

P.T.O.

(7)

- 5. Solve the equations  $\frac{\partial u}{\partial t} = -16 \frac{\partial^2 u}{\partial x^2}$  subjected to the boundary conditions  $u(x,0) = Sin[\pi x]$ ,  $0 \le x \le 1$ , u(0,t) = u(1,t) = 1 using
  - a. Schmidt method.

 $0 \le x \le 1$ .

- b. Crank Nicolson method.
- c. Dufort Frankel method.

Take  $\Delta v = 0.25$ ,  $\Delta t = \frac{1}{36}$ . (14)

- 6. a. A tightly streched string with fixed end points x = 0, and x = 1.0 is at rest in its equilibrium positions. At t = 0, each point of the string is given a velocity 20x(1-x). Find the displacement of the string at x = 0(0.1)1.0 for t = 0(0.1) 1.0 by finite differences using explicit method. Consider the normal form  $\frac{d^2u}{dt^2} = \frac{d^2u}{dx^2}$  for vibrating string.
  - b. Use the method of characteristics to find the solution of the non linear equation  $\frac{\partial^2 u}{\partial x^2} u^2 \frac{\partial^2 u}{\partial y^2} = 0$  at the first characteristic grid point between x = 0.2 and 0.3, y > 0, where u satisfies the condition  $u = 5x^2$ , and  $\frac{\partial u}{\partial y} = 2x$  along the initial line y = 0,
- 7. a. Solve the heat equation  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  subjected to the boundary conditions u(x,0) = 0, u(0,t) = 0, u(1,t) = t. Take  $\Delta t = \frac{1}{36}$ ,  $\Delta x = 0.25$ .
  - b. Obtain the solution of  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = Sin(\pi x).Sin(\pi y)$ ,  $0 \le x, y \le 1$  with u = 0 on the square boundary. Take  $\Delta x = \Delta y = \frac{1}{3}$ . (7)
- 8. Find the solution of  $U_t = U_{xx} + U_{yy}$ ,  $0 \le x, y \le 1$  with conditions  $U(x, y, 0) = Sin(\pi x).Sin(\pi y)$ ,  $U_t(x, y, 0) = 0$ , and U = 0 on the boundary. Take  $\Delta x = \Delta y = \frac{1}{3}, \Delta t = \frac{1}{9}$ . Perform one time integration using first Lees alternating direction implicit method.