



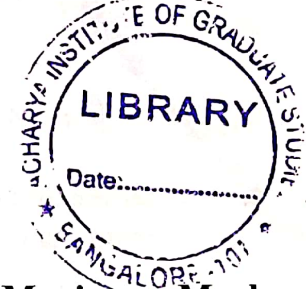
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IV Semester M.Sc. Degree Examination, September/October - 2022

**MATHEMATICS**  
**Graph Theory**  
**(CBCS Scheme Y2K17)**  
**Paper : M-403 T(I)**

**Time : 3 Hours****Maximum Marks : 70****Instructions to Candidates:**

1. Answer any **five (5)** of the following questions.
2. All questions carry **equal** marks.

1. a. Define vertex and edge connectivity in graphs. Show that  $\lambda(K_p) = P - 1$ , for a complete graph of order P.  
b. Let G be a graph of order P with  $P \geq 2$ , and let k be an integer such that  $1 \leq k \leq P - 1$ .  
If  $\deg v \geq \left\lceil \frac{P+k-2}{2} \right\rceil$ , for every vertex v of G, then prove that G is k - connected.  
c. Let u and v be non - adjacent vertices in a graph G. Then prove that the minimum number of vertices in a u - v separating set equals the maximum number of internally disjoint u - v paths in G. (4+4+6)
2. a. Explain planarity of a graph using an example. Write the planarity testing algorithm and check whether the Petersen graph is planar or not applying the algorithm.  
b. Define an outerplanar graph. Prove that a graph is outerplanar if and only if it has no subgraph homeomorphic to  $K_4$  or  $K_{2,3}$ .  
c. Define crossing number  $\nu$  of a graph. Let G be a (p,q) graph. If K is the maximum number of edges in a planar subgraph of G, then prove that  $\nu(G) \geq q - k$ . Further prove that,  $\nu(G) \geq \frac{q^2}{2k} - \frac{q}{2}$ . (5+4+5)
3. a. For any graph G, prove that  $X(G) \leq 1 + \delta(G')$ , where the maximum is taken over all induced subgraphs  $G'$  of G.

**[P.T.O.]**



b. For any graph  $G$ , prove that the sum and product of  $\chi$  and  $\bar{\chi}$  satisfy the following equalities.

i.  $2\sqrt{P} \leq \chi + \bar{\chi} \leq P+1,$

ii.  $P \leq \chi \cdot \bar{\chi} \leq \left(\frac{P+1}{2}\right)^2.$

c. Illustrate the smallest - last sequential algorithm with an appropriate example and find the number of colors required to color the graph. (4+6+4)

4. a. Define edge chromatic index of a graph  $\chi'(G)$ . If  $G$  is a non - empty bipartite graph, then prove that  $\chi'(G) = \Delta(G)$ , where  $\Delta(G)$  is the maximum degree of  $G$ .

b. Briefly discuss 4 - color theorem. Prove that the four colour theorem (conjecture) holds if and only if every cubic bridgeless plane map is 4 - colorable.

c. Establish the chromatic recurrence formula  $\chi(G; k) = \chi(G - e; k) - \chi(G / e; k)$ . Use it to find the chromatic polynomial of  $C_4$ , a cycle on 4 vertices. (4+6+4)

5. a. Define a matching. Prove that every component of the symmetric difference of two matchings is a path or an even cycle.

b. State and prove the König - Egervary theorem.

c. Solve the following assignment problem give its weight matrix using Hungarian algorithm :

$$\begin{bmatrix} 4 & 1 & 6 & 2 & 3 \\ 5 & 0 & 3 & 7 & 6 \\ 2 & 3 & 4 & 5 & 8 \\ 3 & 4 & 6 & 3 & 4 \\ 4 & 6 & 5 & 8 & 6 \end{bmatrix}$$

(3+6+5)

6. a. Define 1 - factor. Prove that a graph  $G$  has a 1 - factor if and only if its order is even and there is no set  $S$  of vertices such that the number of odd components of  $G-S$  exceeds  $|S|$ .

b. Prove that that graph  $K_{2n+1}$  is the sum of  $n$  spanning cycles.

c. Is the following sequence graphical? Provide a construction or a proof of impossibility.

$T = (6, 5, 5, 4, 3, 3, 3, 2, 2).$

(7+3+4)



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7. a. What do you mean by a tournament? Prove that a complete tournament has a directed Hamiltonian path.
- b. Prove that a tournament has a unilateral orientation if and only if  $T$  is a path.
- c. If  $T$  is a tournament with exactly  $k$  strong components, then prove that  $\tilde{T}$  is a transitive tournament of order  $k$ . (6+4+4)
8. a. Explain with an example each, the dominating set, domination number of a graph  $G$ . If a graph  $G$  has  $v(G) \geq 2$ , then prove that  $q \leq \left\lfloor \frac{1}{2}(P - v(G))(P - v(G) + 2) \right\rfloor$ .
- b. For any connected graph  $G$ , prove that  $\left\lceil \frac{\text{diam}(G) + 1}{3} \right\rceil \leq v(G)$ .
- c. Define
- Independent domination number
  - Domestic number, of a graph  $G$ .
- Determine
- $d(\bar{K}_P)$ .
  - $d(C_P)$ .
  - $i(P_P)$ .
  - $i(W_P)$ . (6+4+4)

