60891

Reg. No. [| | | | | | |]

IV Semester M.Sc. Degree Examination, September/October - 2022

MATHEMATICS

Graph Theory

(CBCS Scheme Y2K17)

Paper : M-403 T(I)

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer any **five (5)** of the following questions.
2. All questions carry **equal** marks.

1. a. Define vertex and edge connectivity in graphs. Show that $\lambda(K_P) = P-1$, for a complete graph of order P.

b. Let G be a graph of order P with $P \geq 2$, and let k be an integer such that $1 \leq k \leq P-1$. If $\deg v \geq \left\lceil \frac{P+k-2}{2} \right\rceil$, for every vertex v of G, then prove that G is k - connected.

c. Let u and v be non - adjacent vertices in a graph G. Then prove that the minimum number of vertices in a $u-v$ separating set equals the maximum number of internally disjoint $u-v$ paths in G. **(4+4+6)**

2. a. Explain planarity of a graph using an example. Write the planarity testing algorithm and check whether the petersen graph is planar or not applying the algorithm.

b. Define an outerplanar graph. Prove that a graph is outerplanes if and only if it has no subgraph homeomorphic to K_4 or $K_{2,3}$.

c. Define crossing number v of a graph. Let G be a (p,q) graph. If K is the maximum number of edges in a planar subgraph of G, then prove that $v(G) \geq q-k$. Further prove that, $v(G) \geq \frac{q^2}{2k} - \frac{q}{2}$. **(5+4+5)**

3. a. For any graph G, prove that $X(G) \leq 1 + \delta(G')$, where the maximum is taken over all induced subgraphs G' of G.

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b. For any graph G, prove tha the sum and product of χ and $\overline{\chi}$ satisfy the following in equalities.

i. $2\sqrt{P} \le \chi + \overline{\chi} \le P+1$,

ii. $P \le \chi . \overline{\chi} \le \left(\frac{P+1}{2}\right)^2$.

c. Illustrate the smallest - last sequential algorithm with an appropriate example and find the number of colors required to color the graph. **(4+6+4)**

4. a. Define edge chromatic index of a graph $\chi'(G)$. If G is a non - empty bipartite graph, then prove that $\chi'(G) = \Delta(G)$, where $\Delta(G)$ is the maximum degree of G.

b. Briefly discuss 4 - color theorem. Prove that the four colour theorem (conjecture) holds if and only of every cubic bridgeless plane map is 4 - colorable.

c. Establish the chromatic recurrence formula $\chi(G;k) = \chi(G-e;k) - \chi(G/e;k)$. Use it to find the chromatic polynomial of C_4, a cycle on 4 vertices. **(4+6+4)**

5. a. Define a matching. Prove that every component of the symmetric difference of two matchings is a path or an even cycle.

b. State and prove the König -Egervary theorem.

c. Solve the following assignment problem give its weight matrix using Hungarian algorithm :

$$\begin{bmatrix} 4 & 1 & 6 & 2 & 3 \\ 5 & 0 & 3 & 7 & 6 \\ 2 & 3 & 4 & 5 & 8 \\ 3 & 4 & 6 & 3 & 4 \\ 4 & 6 & 5 & 8 & 6 \end{bmatrix}$$

(3+6+5)

6. a. Define 1 - factor. Prove that a graph G has a 1 - factor if and only if its order is even and there is no set S of vertices such that the number of odd components of G-S exceeds $|S|$.

b. Prove that that graph K_{2n+1} is the sum of n spanning cycles.

c. Is the following sequence graphical? Provide a construction or a proof of impossibility.

$T = (6, 5, 5, 4, 3, 3, 3, 2, 2)$. **(7+3+4)**

7. a. What do you mean by a tournament? Prove that a complete tournament has a directed Hamiltonian path.

b. Prove that a the has a unilateral orientation if and only if T is a path.

c. If T is a tournament with exactly k strong components, then prove that $\tilde{T}$ is a transitive tournament of order k.

(6+4+4)

8. a. Explain with a example each, the dominatng set, domination number of a graph G. If a graph G has $\nu(G) \geq 2$, then prove that $q \leq \left\lfloor \frac{1}{2}(P-\nu(G)).(P-\nu(G)+2) \right\rfloor$.

b. For any connected graph G, prove that $\left\lceil \frac{diam(G)+1}{3} \right\rceil \leq \nu(G)$.

c. Define

i. Independent domination number

ii. Domatic number,

of a graph G.

Determine

i. $d(\overline{k}_P)$.

ii. $d(C_P)$.

iii. $i(P_P)$.

iv. $i(W_P)$.

(6+4+4)