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Reg. No. [ ]

IV Semester M.Sc. Degree Examination, September/October - 2022

MATHEMATICS

Mathematical Methods

(CBCS Y2K17 Scheme)

Paper : M-402 T

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer any **five** questions.
2. All questions carry **equal** marks.

1. a. Solve $u_{xx} - u_{tt} = xt$ with $u = u_t = 0$ when t=0 using Laplace transform.

b. Solve $u_t = u_{xx}, x \geq 0, t \geq 0$ subject to $u(0,t)=0$ and $u(x,0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ by using Fourier sine transform. **(7+7)**

2. a. Solve $u_t = k(u_{rr} + \frac{1}{r}u_r), 0 < r < \infty, t > 0$ with $u(r,0) = f(r)$, for $0 < r < \infty$ where $k > 0$ is a diffusivity constant by using Hankel transform.

b. Solve : $\phi(t) = \sin t - 2\int_0^1 \cos(t-x)\phi(x)dx$. **(7+7)**

3. a. Convert $y'' - 3y' + 2y = 4\sin x$ with $y(0) = 1$, $y'(0) = -2$ into the corresponding integral equation. Conversely derive the original differential equation along with initial conditions from the integral equation so obtained.

b. Solve $u(x) = \cos x - x - 2 + \int_0^x (t-x)u(t)dt$ by the method of Resolvent kernel. **(7+7)**

4. a. Solve $u(x) = x + \lambda\int_0^{2\pi} |\pi - t| \sin x u(t)dt$ by using the method of degenerate Kernel.

b. Solve $u(x) = x^2 + 1 + \frac{3}{2}\int_{-1}^{1} (xt + x^2t^2)u(t)dt$ by using the Hilbert method. **(7+7)**

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5. a. Find the asymptotic behaviour of $I(x)=\int_0^x t^{\frac{-1}{2}} e^{-t} dt$ as $x \to \infty$.

b. Use Laplace method to find the leading behavior of $\int_0^{\frac{\pi}{2}} e^{-x} \tan t \, dt$ as $x \to \infty$. **(7+7)**

6. a. Find the asymptotic expansion of $I(x)=\int_0^{\frac{\pi}{2}} e^{-t} t^{x-1} dt$ as $x \to \infty$ by using Watson lemma.

b. Use the method of steepest decent to find the asymptotic expansion of $I(x)=\int_0^1 \log t e^{ixt} dt$. **(7+7)**

7. a. Apply the perturbation method to find the approximate solution of $y''=-e^{-x}y$ with y(0) = 1, $y'(0)=1$.

b. Find the two - term periodic solution of $y''+y=\in\left(y'-\frac{1}{3}y'^3\right)$ with $y(0)=A$, $y'(0)=0$.

(7+7)

8. a. Solve $\in y''+(1+\in)y'+y=0$ with $y(0)=0$, $y(1)=1$ by using singular perturbation method.

b. Find the leading term of WKB approximate solution of $\in 62 y''=Q(x)y$ where $Q(x)\neq 0$. **(7+7)**
