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IV Semester M.Sc. Degree Examination, September/October - 2022

MATHEMATICS

Mathematical Methods

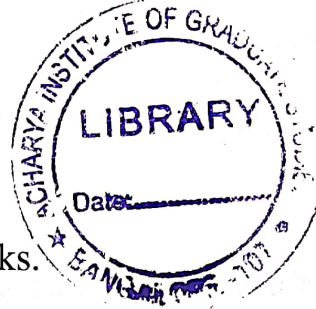
(CBCS Y2K17 Scheme)

Paper : M-402 T

Time : 3 Hours

Instructions to Candidates:

1. Answer any **five** questions.
2. All questions carry **equal** marks.



Maximum Marks : 70

1. a. Solve $u_{xx} - u_t = xt$ with $u = u_t = 0$ when $t = 0$ using Laplace transform.
- b. Solve $u_t = u_{xx}, x \geq 0, t \geq 0$ subject to $u(0, t) = 0$ and $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ by using Fourier sine transform. (7+7)
2. a. Solve $u_t = k(u_{rr} + \frac{1}{r}u_r), 0 < r < \infty, t > 0$ with $u(r, 0) = f(r)$, for $0 < r < \infty$ where $k > 0$ is a diffusivity constant by using Hankel transform.
- b. Solve : $\phi(t) = \sin t - 2 \int_0^1 \cos(t-x)\phi(x)dx$. (7+7)
3. a. Convert $y'' - 3y' + 2y = 4 \sin x$ with $y(0) = 1, y'(0) = -2$ into the corresponding integral equation. Conversely derive the original differential equation along with initial conditions from the integral equation so obtained.
- b. Solve $u(x) = \cos x - x - 2 + \int_0^x (t-x)u(t)dt$ by the method of Resolvent kernel. (7+7)
4. a. Solve $u(x) = x + \lambda \int_0^{2\pi} |\pi - t| \sin xu(t)dt$ by using the method of degenerate Kernel.
- b. Solve $u(x) = x^2 + 1 + \frac{3}{2} \int_{-1}^1 (xt + x^2 t^2)u(t)dt$ by using the Hilbert method. (7+7)

[P.T.O.]



5. a. Find the asymptotic behaviour of $I(x) = \int_0^x t^{-\frac{1}{2}} e^{-t} dt$ as $x \rightarrow \infty$.
- b. Use Laplace method to find the leading behavior of $\int_0^{\frac{\pi}{2}} e^{-x} \tan t dt$ as $x \rightarrow \infty$. (7+7)
6. a. Find the asymptotic expansion of $I(x) = \int_0^{\frac{\pi}{2}} e^{-t} t^{x-1} dt$ as $x \rightarrow \infty$ by using Watson lemma.
- b. Use the method of steepest descent to find the asymptotic expansion of $I(x) = \int_0^1 \log t e^{ixt} dt$. (7+7)
7. a. Apply the perturbation method to find the approximate solution of $y'' = -e^{-x}y$ with $y(0) = 1, y'(0) = 1$.
- b. Find the two-term periodic solution of $y'' + y = \epsilon \left(y' - \frac{1}{3} y^3 \right)$ with $y(0) = A, y'(0) = 0$. (7+7)
8. a. Solve $\epsilon y'' + (1 + \epsilon)y' + y = 0$ with $y(0) = 0, y(1) = 1$ by using singular perturbation method.
- b. Find the leading term of WKB approximate solution of $\epsilon y'' = Q(x)y$ where $Q(x) \neq 0$. (7+7)

