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IV Semester M.Sc. Degree Examination, September/October - 2022

MATHEMATICS

Measure and Integration

(CBCS - Y2K17 Scheme)

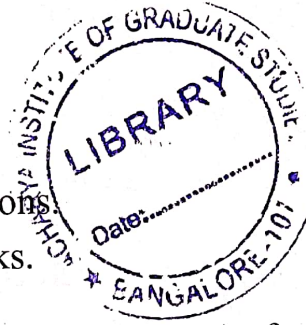
Paper M 401 T

Time : 3 Hours

Instructions to Candidates:

- i. Answer any **five (5)** full questions.
- ii. All questions carry **equal** marks.

Maximum Marks : 70



1. a. Define Lebesgue outer measure m^*A of a set A of real numbers. Let $\{A_n\}$ be a countable collection of sets of real numbers. Prove that $m^*(\cup A_n) \leq \sum m^*A_n$.
- b. Show that collection of all measurable sets is a σ -algebra.
- c. If $\{I_n\}$ is a finite covering of $Q \cap [0,1]$ by open intervals, show that $\sum I(I_n) \geq 1$. Is this true if $\{I_n\}$ is infinite? Justify. (5+4+5)
2. a. Show that a set $E \subset R^1$ is measurable if and only if given $\varepsilon > 0, \exists$ a finite union I of intervals such that $m^*(E \Delta I) < \varepsilon$.
- b. Construct an uncountable set of measure zero. (7+7)
3. a. State Littlewood's three principles and prove any one of them.
- b. Let $f \geq 0$ and measurable. Show that there exists a sequence $\{\varphi_n\}$ of simple functions such that $\varphi_n \uparrow f$. (6+8)
4. a. State and prove Fatou's lemma.
- b. Let $\int_F f < \infty, \int_E g < \infty$. Show that $\int_E (f+g) = \int_E f + \int_E g$. (7+7)
5. a. State and prove Vitali Covering Lemma.
- b. If f is integrable on $[a,b]$ and $\int_a^b f(t) dt = 0$ for all $t \in [a,b]$ then show that $f(t) = 0$ a.e. on $[a,b]$. (7+7)

[P.T.O.]



6. a. Find the Dini Derivatives of $f(x) = \begin{cases} 0 & x = 0 \\ x \sin \frac{1}{x} & x \neq 0 \end{cases}$ at $x = 0$.
- b. If $\int_a^b f < \infty$ and $F(x) = F(a) + \int_a^x f(t)dt$, show $F' = f$ a.e. on $[a,b]$. (7+7)
7. a. With usual notation prove that $T = P+N$ and $f(b)-f(a) = P-N$.
- b. If f exists and bounded on $[a,b]$, then prove that f is of bounded variation on $[a,b]$.
- c. Show that $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & 0 \leq x \leq 1 \\ 0 & x = 0 \end{cases}$ is of bounded variation on $[0,1]$. (5+4+5)
8. a. If $f_1, f_2 : [a,b] \rightarrow \mathbb{R}$ are absolutely continuous, Is $f_1 \circ f_2$ absolutely continuous on $[a,b]$?
- b. If f is absolutely continuous on $[a,b]$, show $f \in BV[a,b]$ and that f is an N function.
- c. Show that a function f is an indefinite integral if and only if it is absolutely continuous. (3+4+7)

