60881

Reg. No.

IV Semester M.Sc. Degree Examination, September/October - 2022

MATHEMATICS

Measure and Integration

(CBCS - Y2K17 Scheme)

Paper M 401 T

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

i. Answer any **five (5)** full questions.

ii. All questions carry **equal** marks.

1. a. Define Lebesgue outer measure m^*A of a set A of real numbers. Let $\{A_n\}$ be a countable collection of sets of real numbers. Prove that $m^*(\cup A_n) \le \sum m^* A_n$.

   b. Show that collection of all measurable sets is a σ - algebra.

   c. If $\{I_n\}$ is a finite covering of $Q \cap [0,1]$ by open intervals, show that $\sum I(I_n) \ge 1$. Is this true if $\{I_n\}$ is infinite? Justify. **(5+4+5)**

2. a. Show that a set $E \subset R^1$ is measurable if and only if given $\varepsilon > 0, \exists$ a finite union I of intervals such that $m^*(E \Delta I < \varepsilon)$.

   b. Construct an uncountable set of measure zero. **(7+7)**

3. a. State Littlewood's three principles and prove any one of them.

   b. Let $f \ge 0$ and measurable. Show that there exists a sequence $\{\varphi_n\}$ of simple functions such that $\varphi_n \uparrow f$. **(6+8)**

4. a. State and prove Fatou's lemma.

   b. Let $\int_F f < \infty, \int_E g < \infty$. Show that $\int_E (f+g) = \int_E f + \int_E g$. **(7+7)**

5. a. State and prove Vitali Covering Lemma.

   b. If f is integrable on $|a,b|$ and $\int_a^b f(t)dt = 0$ for all $t \in [a,b]$ then show that $f(t) = 0$ a.e. on $|a,b|$. **(7+7)**

|P.T.O.

6. a. Find the Dini Derivatives of $f(x)=\begin{cases} 0 & x=0 \\ x\,Sin\frac{1}{x} & x\neq 0 \end{cases}$ at x = 0.

b. If $\int_a^b f<\infty$ and $F(x)=F(a)+\int_a^x f(t)dt$, show $F'=f$ a.e. on [a,b]. **(7+7)**

7. a. With usual notation prove that $T=P+N$ *and* $f(b)\text{-}f(a)=P\text{-}N$.

b. If f exists and bounded on [a,b], then prove that f is of bounded variation on [a,b].

c. Show that $f(x)=\begin{cases} x^2\sin\frac{1}{x} & 0\leq x\leq 1 \\ 0 & x=0 \end{cases}$ is of bounded variation on [0,1]. **(5+4+5)**

8. a. If $f_1, f_2 : [a,b]\rightarrow |a,b|$ are absolutely continuous, If $f_1\, o\, f_2$ absolutely continuous on [a,b]?

b. If f is absolutely continuous on [a,b], show $f\in BV[a,b]$ and that f is an N function.

c. Show that a function f is an indefinite integral if and only if it is absolutely continuous. **(3+4+7)**
