

MAKE-UP EXAM

USN

BMATE201

Second Semester B.E/B.Tech. Degree Examination, Nov./Dec. 2023

Mathematics - II for EEE Stream

Time: 3 hrs.

Max. Marks:100

- Note:** 1. Answer any FIVE full questions, choosing ONE full question from each module.
 2. M : Marks , L: Bloom's level , C: Course outcomes.
 3. VTU formula hand book is permitted.

		Module - 1	M	L	C
1	a.	If $\phi = x^2 + y - z - 1$ find grad ϕ at $(1, 0, 0)$. Also find its magnitude.	6	L3	CO1
	b.	Find the divergence and curl of the vector : $\vec{F} = (xyz) \hat{i} + (3x^2y) \hat{j} + (xz^2 - y^2z) \hat{k}$ at $(2, -1, 1)$.	7	L2	CO1
	c.	Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x) \hat{i} + (3xz + 2xy) \hat{j} + (3xy - 2xz + 2z) \hat{k}$ is both solenoidal and irrotational.	7	L2	CO1

OR

2	a.	Suppose $\vec{F} = x^3 \hat{i} + y \hat{j} + z \hat{k}$ is the force field. Find the work done by \vec{F} along the line from $(1, 2, 3)$ to $(3, 5, 7)$.	6	L2	CO1
	b.	Verify Green's theorem in the xy -plane for $\int_C (xy + y^2) dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.	7	L3	CO1
	c.	Using modern mathematical tools, write the code to find the divergence of $\vec{F} = x^2y \hat{i} + yz^2 \hat{j} + x^2z \hat{k}$.	7	L3	CO5

Module - 2

3	a.	Define a subspace. Show that a subset $S = \{x_1, x_2, x_3 \mid x_1 + x_2 + x_3 = 0\}$ of $V_3(\mathbb{R})$ is a subspace of $V_3(\mathbb{R})$.	6	L2	CO2
	b.	Prove that in $V_3(\mathbb{R})$ the vectors $\{(1, 2, 1), (3, 1, 5), (3, -4, 7)\}$ are linearly independent.	7	L2	CO2
		Find $\langle p, q \rangle$ and $\ P\ $. Given $P(x) = x^2 - x$, $q(x) = x + 1$, the inner product space $\langle p, q \rangle = \int_{-1}^1 P(x); q(x) dx$.	7	L2	CO2

OR

4	a.	Let $T : U \rightarrow V$ be a linear transformation defined by, $T(x, y, z) = \{(x + y, x - y, 2x + z)/x, y, z, \in R\}$. Verify Rank Nullity theorem.	6	L2	CO2
	b.	Explain the vector $(2, -5, -1)$ as a linear combination of the vectors $(1, 2, 3)(2, 1, 1)(1, 3, 2)$ of $V_3(R)$.	7	L2	CO2
	c.	Using the modern mathematical tool write the code to represent the reflection transformation $T : R^L \rightarrow R^2$ and to find the image of vector $(10, 0)$ when it reflected about the y -axis.	7	L3	CO5

Module - 3

5	a.	Find the Laplace transform of $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$.	6	L2	CO3
	b.	Find $L^{-1}\left\{\frac{1}{s(s^2+1)}\right\}$ using convolution theorem.	7	L2	CO3
	c.	Express $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \cos 2t & \pi < t < 2\pi \\ \cos 3t & t > 2\pi \end{cases}$ in terms of unit step function and find $L\{f(t)\}$.	7	L3	CO3

OR

6	a.	Find the inverse Laplace transform of i) $\frac{(s+2)^3}{s^6}$ ii) $\frac{2s+5}{4s^2+25}$.	6	L2	CO3
	b.	Solve by Laplace transform method : $y'' + 4y' + 3y = e^{-t}$; $y(0) = 1 = y'(0)$.	7	L2	CO3
	c.	Find the Laplace transform of the square wave function of period a , defined by $f(t) = \begin{cases} K & 0 < t < a/2 \\ -K & a/2 < t < a \end{cases}$	7	L2	CO3

Module - 4

7	a.	Evaluate $\int_2^7 \left(\frac{1}{x}\right) dx$, using Trapezoidal rule, taking $n = 5$.	6	L3	CO4														
	b.	Find the real root of the equation $e^x - 3x - \sin x = 0$ by the Regula - Falsi method between 0 and 1. (carry out three iterations) x is in radians.	7	L2	CO4														
	c.	Find y at $x = 1$ using Newton divided difference formula for the following data : <table border="1" style="margin-left: auto; margin-right: auto;"><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>4</td><td>7</td><td>9</td></tr><tr><td>y</td><td>4</td><td>26</td><td>58</td><td>112</td><td>466</td><td>922</td></tr></table>	x	0	2	3	4	7	9	y	4	26	58	112	466	922	7	L2	CO4
x	0	2	3	4	7	9													
y	4	26	58	112	466	922													

OR

8	a.	Evaluate : $\int_2^{\pi/2} \cos x dx$, using Simpson's $(1/3)^{rd}$ rule with $n = 8$ [x in radian].	6	L3	CO4												
	b.	Construct Newton's forward interpolation polynomial for the data :	7	L2	CO4												
	c.	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>f(x)</td><td>3</td><td>6</td><td>11</td><td>18</td><td>27</td></tr> </table>	x	0	1	2	3	4	f(x)	3	6	11	18	27	7	L2	CO4
x	0	1	2	3	4												
f(x)	3	6	11	18	27												

Module - 5

9	a.	Using Taylor's method to find $y(0.2)$ by considering the terms upto 4^{th} degree, given $\frac{dy}{dx} - 2y - 3e^x = 0$; $y(0) = 0$.	6	L3	CO4
	b.	Given $\frac{dy}{dx} = x + y$; $y(0) = 1$. Compute $y(0.2)$ using Runge - Kutta 4^{th} order method [$h = 0.2$].	7	L2	CO4
	c.	Apply Milne's predictor and corrector method find y at $x = 2$ given $\frac{dy}{dx} = \frac{2y}{x}$ ($x \neq 0$)	7	L2	CO4

x	1	1.25	1.5	1.75
y	2	3.13	4.5	6.13

OR

10	a.	Using Modified Euler's method to find y at $x = 0.2$ given $y' = \frac{x-y}{2}$; $y(0) = 1$ [$h = 0.1$].	6	L3	CO4
	b.	Find $y(1.1)$ by using Runge-Kutta method of fourth order. Given $\frac{dy}{dx} = x(y)^{1/3}$; $y(1) = 1$ [take $h = 0.1$].	7	L2	CO4
	c.	Using modern mathematical tools, write a code to find $y(0.1)$ given $\frac{dy}{dx} = x - y$, $y(0) = 1$, by Taylor's series.	7	L3	CO5

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