CBCS SCHEME

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18EC54

Fifth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Information Theory and Coding

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive an expression for average information content of symbols in long independent sequences. **(04 Marks)**

b. Find the relationship between Hartleys, nats and bits. **(06 Marks)**

c. For the Markov source of Fig. Q1 (c), find
(i) Entropy of each state.
(ii) Entropy of the source.
(iii) G_1, G_2. Also show that $G_1 > G_2 > H(s)$. **(10 Marks)**

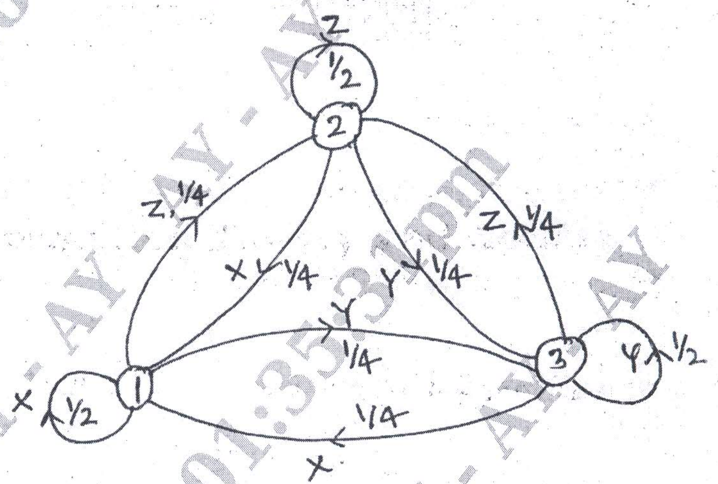

Fig. Q1 (c)

OR

2 a. A binary source is emitting an independent sequence of 0's and 1's . With probabilities 'P' and '1 – P' respectively. Plot the entropy of the source versus 'P'. **(04 Marks)**

b. A code is composed of dots and dashes. Assuming that a dash is 3 times as long as a dot and has one-third the probability of occurrence, calculate
(i) The information in a dot and a dash.
(ii) The entropy of dot-dash code.
(iii) The average rate of information if a dot lasts for 10 m-sec and this time is allowed between symbols. **(08 Marks)**

c. Consider the state diagram of Markov source of Fig. Q2 (c).
   (i) Compute the state probabilities
   (ii) Find entropy of each state.
   (iii) Find the entropy of the source. **(08 Marks)**

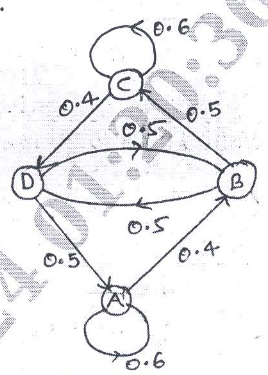

Fig. Q2 (c)

Module-2

3 a. Apply Shannon's encoding (binary) algorithm to the following set of messages and obtain code efficiency and redundancy.

m_1	m_2	m_3	m_4	m_5
$\frac{1}{8}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{8}$

(10 Marks)

b. A discrete memoryless source has an alphabet of seven symbols with probabilities for its output, as described below.

Symbol	S_0	S_1	S_2	S_3	S_4	S_5	S_6
Probability	0.25	0.25	0.125	0.125	0.125	0.0625	0.0625

Compute Shannon-Fano code for this source. Find coding efficiency. **(10 Marks)**

OR

4 a. Consider a zero-memory source with
$S = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$, $P = \{0.4, 0.2, 0.1, 0.1, 0.1, 0.05, 0.05\}$
   (i) Construct a binary Huffman code by placing the composite symbol as low as possible. Find the coding efficiency.
   (ii) Repeat (i) by moving the composite symbol as high as possible and find the coding efficiency. **(12 Marks)**

b. Explain the properties of codes. Also draw the code-property circle diagram. **(08 Marks)**

Module-3

5 a. A binary symmetric channel has the following noise matrix with source probabilities of $P(X_1) = \frac{2}{3}$ and $P(X_2) = \frac{1}{3}$,

$$p\left(\frac{Y}{X}\right) = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

   (i) Determine H(x), H(y), H(x, y), H(Y/X), H(X/Y) and I(X, Y).
   (ii) Find the channel capacity.
   (iii) Find channel efficiency and redundancy. **(08 Marks)**

b. What is mutual information? Explain its properties. **(06 Marks)**

c. Consider the Binary Symmetric channel with channel matrix given by,

$$p\left(\frac{Y}{X}\right) = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Find the channel capacity using Muroga's method. **(06 Marks)**

OR

6 a. Explain the following :
   (i) Symmetric / Uniform channels.
   (ii) Binary symmetric channels.
   (iii) Binary Erasure channels. **(06 Marks)**

b. An analog signal has a 4 kHz bandwidth. The signal is sampled at 2.5 times the Nyquist rate and each sample quatized into 256 equally likely levels. Assume that the successive samples are statistically independent.
   (i) Find the information rate of this source.
   (ii) Can the output of this source be transmitted without errors over Gaussian channel of bandwidth 50 kHz and the signal to noise ratio (SNR) of 20 dB?
   (iii) If the output of this source is to be transmitted without errors over an analog channel having (S/N) of 10 dB, compute the bandwidth requirement of the channel. **(08 Marks)**

c. State Shannon-Hartley law. Explain the implications of Shannon-Hartley law. **(06 Marks)**

Module-4

7 a. Define (i) Hamming weight (ii) Hamming distance (iii) Minimum distance **(06 Marks)**

b. For a systematic (6, 3) Linear Block Code, $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$
   (i) Find all the code vectors.
   (ii) Draw the encoder circuit.
   (iii) Find minimum distance. **(08 Marks)**

c. For a systematic (7, 4) Linear Block Code, the parity matrix 'P' is given as,

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

   (i) Draw the encoder circuit.
   (ii) A single error has occurred in each of these received vectors. Detect and correct errors.
      (a) $R_A = [0111110]$
      (b) $R_B = [1011100]$ **(06 Marks)**

OR

8 a. The parity checkbits of a (7, 4) Hamming code are generated by,

$C_5 = d_1 + d_3 + d_4$

$C_6 = d_1 + d_2 + d_3$

$C_7 = d_2 + d_3 + d_4$

where d_1, d_2, d_3 and d_4 are message bits.

(i) Find the generator matrix 'G' and the parity check matrix 'H' of this code.
(ii) Draw the syndrome calculation circuit for this code.
(iii) Check for $GH^T = 0$ **(08 Marks)**

b. A (15, 5) linear cyclic code has generator polynomial $g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$,
(i) Draw the encoder for this code.
(ii) Tabulate the contents of shift registers of the encoder for the message polynomial, $D(x) = 1 + x^2 + x^4$ **(06 Marks)**

c. Design an encoder for the (7, 4) cyclic code generated by $g(x) = 1+x+x^3$ and verify its operation using the message vectors (1 0 0 1) and (1 0 1 1). **(06 Marks)**

Module-5

9 a. Consider the (3, 1, 2) convolutional code with $g^{(1)} = (1\ 1\ 0)$, $g^{(2)} = (1\ 0\ 1)$ and $g^{(3)} = (1\ 1\ 1)$.
(i) Draw the encoder block diagram.
(ii) Find the generator matrix.
(iii) Find the code word corresponding to the information sequence (1 1 1 0 1) using time-domain and transform-domain approach. **(12 Marks)**

b. For the (2, 1, 2) convolutional encoder, described by $g^{(1)} = (1\ 1\ 1)$ and $g^{(2)} = (1\ 0\ 1)$,
(i) Draw the encoder circuit.
(ii) Find the generator matrix.
(iii) Find the output sequence for the information sequence d = 1 0 0 1 1 using time domain and transform-domain approach. **(08 Marks)**

OR

10 a. For the (2, 1, 2) convolutional encoder described by $g^{(1)} = (1\ 1\ 1)$ and $g^{(2)} = (1\ 0\ 1)$,
(i) Draw the state table.
(ii) Write the state transition table.
(iii) Draw the state diagram. **(06 Marks)**

b. For the (3, 1, 2) convolutional encoder, with $g^{(1)} = (1\ 1\ 0)$, $g^{(2)} = (1\ 0\ 1)$ and $g^{(3)} = (1\ 1\ 1)$.
(i) Draw state table, state transition table and state diagram.
(ii) Construct the code-tree and find the output sequence for the message sequence (1 1 1 0 1) **(14 Marks)**

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