CBCS SCHEME

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18MAT31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the Laplace transform of i) $e^{-t}\sin 3t$ ii) $\frac{\cos at - \cos bt}{t}$. **(07 Marks)**

b. The square wave function f(t) with period "a" is defined by

$$f(t) = \begin{cases} E, & 0 \le t < a/2 \\ -E, & a/2 \le t < a \end{cases}$$

Show that $L\{f(t)\} = (E/s)\tanh(as/4)$. **(06 Marks)**

c. Using Laplace transform method to solve $y'' - 3y' + 2y = e^{3t}$, $y(0) = 1$ and $y'(0) = 0$. **(07 Marks)**

OR

2 a. Find i) $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$ ii) $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$. **(07 Marks)**

b. Find the Laplace transform of $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$ by using unit – step function. **(06 Marks)**

c. Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$, using Convolution theorem. **(07 Marks)**

Module-2

3 a. Find a Fourier series to represent $(x - x^2)$ from $x = -\pi$ to $x = \pi$. **(07 Marks)**

b. Find the half – range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. **(06 Marks)**

c. The following table gives the variations of periodic current over a period :

t(sec) :	0	T/6	T/3	T/2	2T/3	5T/6	T
A(amp) :	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic.. **(07 Marks)**

OR

4 a. Obtain Fourier series for the function

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots = \frac{\pi^2}{8}$. **(07 Marks)**

b. Find the half range Fourier sine series of $f(x) = x(\pi - x)$, $0 \le x \le \pi$. **(06 Marks)**

c. Obtain Fourier series for the function f(x), given by $f(x) = \begin{cases} 1 + \frac{2x}{\pi} , & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} , & 0 \le x \le \pi \end{cases}$. **(07 Marks)**

Module-3

5 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 , & |x| \le 1 \\ 0 , & |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right) \cos (x/2)\, dx$. **(07 Marks)**

b. Find the Fourier sin transform of e^{-ax} , $a > 0$. **(06 Marks)**

c. Solve $u_{n+2} + 6\,u_{n+1} + 9\,u_n = 2^n$ with $u_0 = u_1 = 0$ by using Z – transforms. **(07 Marks)**

OR

6 a. Find the Fourier transform of

$f(x) = \begin{cases} 1 , & |x| < 1 \\ 0 , & |x| > 1 \end{cases}$ Hence evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$. **(07 Marks)**

b. Find the Z – transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. **(06 Marks)**

c. Find the Inverse Z – transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(07 Marks)**

Module-4

7 a. Employ Taylor's method to obtain approximate value of y at x = 0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$. **(07 Marks)**

b. Solve the differential equation $\frac{dy}{dx} = xy^2$ under the initial condition y(0) = 1 by using modified Euler's method at the point x = 0.05 (h = 0.05). **(07 Marks)**

c. Apply Milne's predictor – corrector formulae to compute y(1.4) correct to four decimal places. Given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data y(1) = 2 , y(1.1) = 2.2156 , y(1.2) = 2.4649 , y(1.3) = 2.7514. **(06 Marks)**

OR

8 a. Using fourth order Runge Kutta method, compute y(0.2). Given that $\frac{dy}{dx} = 3x + \frac{y}{2}$, y(0) = 1 (take h = 0.2). **(07 Marks)**

b. Using modified Euler's method to find y(0.2) correct to four decimals by solving the equation $\frac{dy}{dx} = x - y^2$, y(0) = 1 by taking h = 0.1. (Perform 2 iterations in each step). **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1+y)$ and y(1) = 1 , y(1.1) = 1.2330 , y(1.2) = 1.5480 , y(1.3) = 1.9790. Evaluate y(1.4) by Adms – Bashforth method. **(06 Marks)**

Module-5

9 a. Using fourth order Runge – Kutta method solve $y'' = x(y')^2 - y^2$ for x = 0.2 correct to four decimal places. Initial conditions are x = 0 , y = 1 and y′ = 0. **(07 Marks)**

b. Solve the variational problem $\delta\int_0^1 \{x + y + (y')^2\} dx = 0$ under the conditions y(0) = 1 and y(1) = 2. **(07 Marks)**

c. With usual notation prove that $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

OR

10 a. Apply Milne's method to compute y(0.4) given $y'' + xy' + y = 0$, y(0) = 1 , y(0.1) = 0.995 y(0.2) = 0.9802 , y(0.3) = 0.956 and y′(0) = 0 , y′(0.1) = - 0.0995 , y′(0.2) = - 0.196 , y′(0.3) = - 0.2863. **(07 Marks)**

b. Solve the variational problem $\delta\int_0^{\pi/2} \{y^2 - (y')^2\} dx = 0$, y(0) = 0 , y(π/2) = 2. **(07 Marks)**

c. Prove that the geodesics on a plane are straight lines. **(06 Marks)**

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