

# CBCS SCHEME



BMATE301/BEE301

## Third Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024 Mathematics-III for EE Engineering

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. VTU Formula Hand Book is permitted.  
3. M : Marks , L: Bloom's level , C: Course outcomes.  
4. Mathematics handbook is permitted.*

Module – 1			M	L	C																			
Q.1	a.	Solve : $(D^4 + 8D^2 + 16)y = 0$ .	6	L1	CO1																			
	b.	Solve : $(D^3 - 3D + 2)y = 2 \sinh x$	7	L2	CO1																			
	c.	Solve : $x^2 y'' - 3xy' + 5y = 3 \sin(\log x)$	7	L3	CO1																			
<b>OR</b>																								
Q.2	a.	Solve : $(D^4 - 4D^3 - 5D^2 - 36D - 36)y = 0$ .	6	L1	CO1																			
	b.	Solve : $\frac{d^3 y}{dx^3} + 2 \frac{d^2 y}{dx^2} + \frac{dy}{dx} = \sin 2x$ .	7	L2	CO1																			
	c.	Solve : $(2x + 1)^2 \frac{d^2 y}{dx^2} - 2(2x + 1) \frac{dy}{dx} - 12y = 3(2x + 1)$ .	7	L3	CO1																			
<b>Module – 2</b>																								
Q.3	a.	Find the curve at best fit of the form $y = ax^6$ to the following data : <table border="1" style="display: inline-table; margin: 5px;"> <tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>y</td><td>0.5</td><td>2</td><td>4.5</td><td>8</td><td>12.5</td></tr> </table>	x	1	2	3	4	5	y	0.5	2	4.5	8	12.5	6	L2	CO2							
	x	1	2	3	4	5																		
	y	0.5	2	4.5	8	12.5																		
b.	Calculate the coefficient of correlation and obtain the lines of regression for the following data : <table border="1" style="display: inline-table; margin: 5px;"> <tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr> <tr><td>y</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td><td>16</td><td>15</td></tr> </table>	x	1	2	3	4	5	6	7	8	9	y	9	8	10	12	11	13	14	16	15	7	L3	CO2
x	1	2	3	4	5	6	7	8	9															
y	9	8	10	12	11	13	14	16	15															
c.	In a partially destroyed laboratory record of correlation data, following results only available : Variance of $x$ is 9 and regression lines, $4x - 5y + 33 = 0$ ; $20x - 9y = 107$ . Find (i) Mean value of $x$ and $y$ (ii) SD of $y$ . (iii) Coefficient of correlation between $x$ and $y$ .	7	L4	CO2																				
<b>OR</b>																								
Q.4	a.	Fit a curve of the form, $y = ax^2 + bx + c$ to the following data : <table border="1" style="display: inline-table; margin: 5px;"> <tr><td>x :</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>y :</td><td>10</td><td>12</td><td>8</td><td>10</td><td>14</td></tr> </table>	x :	1	2	3	4	5	y :	10	12	8	10	14	6	L2	CO2							
	x :	1	2	3	4	5																		
y :	10	12	8	10	14																			
b.	If $\theta$ is the acute angle between the two regression lines relating the variables $x$ and $y$ , show that $\tan \theta = \left( \frac{1-r^2}{r} \right) \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$ . Indicate the significance of the cases $r = 0$ and $r = \pm 1$	7	L2	CO2																				

c.	Ten competitor's in a music contest ranked by 3 judges A, B, C in the following order. Use the rank correlation coefficient to decide which pair judges have the nearest approach to common test of music.	7	L3	CO2
A	1 6 5 10 3 2 4 9 7 8			
B	3 5 8 4 7 10 2 1 6 9			
C	6 4 9 8 1 2 3 10 5 7			

**Module - 3**

Q.5	a.	Find the Fourier series for the function $f(x) = x^2$ in the interval $-\pi \leq x \leq \pi$ , hence deduce the $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ .	6	L2	CO3
	b.	Expand the function $f(x) = x(\pi - x)$ over the interval $(0, \pi)$ in half range cosine Fourier series hence deduce that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$	7	L3	CO3

c.	The following table gives the variations of a periodic current A over a certain period T.	7	L3	CO3			
t(sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A(amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a current part of 0.75 amp in the current A and obtain the amplitude of the first harmonic.

**OR**

Q.6	a.	Find the Fourier expansion of the function $f(x) = (\pi - x)^2$ over the interval $0 \leq x \leq 2\pi$ . Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ .	7	L2	CO3
	b.	Expand the function $f(x) = \begin{cases} \frac{1}{4} - x & \text{for } 0 < x < \frac{1}{2} \\ 0 - \frac{3}{4} & \frac{1}{2} < x < 1 \end{cases}$ in the half range sine series.	6	L2	CO3

c.	Find the constant term and the first harmonic in the Fourier series for f(x) given by the table.	7	L3	CO3			
x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\pi$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$2\pi$
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

**Module - 4**

Q.7	a.	Find the Fourier transform of the function, $f(x) = \begin{cases} 1 & \text{for }  x  \leq a \\ 0 & \text{for }  x  \geq a \end{cases}$  where a is a positive constant hence evaluate integrals , $\int_{-\infty}^{\infty} \frac{\sin ax \cos ax}{x} dx$	6	L2	CO4
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	b.	Find the Fourier cosine transform of $f(x) = e^{-ax}$ , $a > 0$ , hence deduce that $\int_0^{\infty} \frac{\cos mx}{x^2 + a^2} dx = \frac{\pi}{2a} e^{-am}$	7	L3	CO4																
	c.	Find the inverse z-transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$ .	7	L3	CO4																
<b>OR</b>																					
Q.8	a.	Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$ .	6	L2	CO4																
	b.	Find the z-transform of $\sin n\theta$ and $\cos n\theta$ hence find $z\left\{\cos\left(\frac{n\pi}{2}\right)\right\}$ and $z\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$ .	7	L3	CO4																
	c.	Solve the difference equation, $u_{n+2} - 5u_{n+1} + 6u_n = 2$ given $u_0 = 3$ , $u_1 = 7$ , using z-transforms.	7	L3	CO4																
<b>Module – 5</b>																					
Q.9	a.	Define (i) Type I and Type II errors. (ii) Confidence interval. (iii) Level of significance.	6	L1	CO5																
	b.	The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that (i) Exactly 2 will be defective. (ii) At least 2 will be defective. (iii) None will be defective	7	L2	CO5																
	c.	In normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and SD, given that $A(0.5) = 0.19$ and $A(1.4) = 0.42$ , where $A(Z)$ is the area under the standard normal curve from 0 to z.	7	L3	CO5																
<b>OR</b>																					
Q.10	a.	The pdf $P(x)$ of a variate $X$ is given by the table : <table border="1" style="margin-left: 20px;"> <tr> <td>x :</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>P(x) :</td> <td>K</td> <td>3K</td> <td>5K</td> <td>7K</td> <td>9K</td> <td>11K</td> <td>13K</td> </tr> </table> For what value of K, does this represent a valid probability distribution? Also find $P(x < 4)$ , $P(x \geq 5)$ and $P(3 < x \leq 6)$ .	x :	0	1	2	3	4	5	6	P(x) :	K	3K	5K	7K	9K	11K	13K	6	L2	CO5
x :	0	1	2	3	4	5	6														
P(x) :	K	3K	5K	7K	9K	11K	13K														
	b.	Consider the sample consisting of nine numbers, 45, 47, 50, 52, 48, 47, 49, 53 and 51. The sample is drawn from a population whose mean is 47.5. Find whether the sample mean differs significantly from the population mean at 5% level of significance (Given $t_{0.05}(df = 8) = 2.31$ )	7	L3	CO5																
	c.	A die is thrown 60 times and the frequency distribution for the number appearing on the face $x$ is given by the table : <table border="1" style="margin-left: 20px;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>frequency</td> <td>15</td> <td>6</td> <td>4</td> <td>7</td> <td>11</td> <td>17</td> </tr> </table> Test the hypothesis that the die is unbiased. Given $\chi_{0.05}^2(5) = 11.07$ and $\chi_{0.01}^2(5) = 15.09$ .	x	1	2	3	4	5	6	frequency	15	6	4	7	11	17	7	L3	CO5		
x	1	2	3	4	5	6															
frequency	15	6	4	7	11	17															

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