CBCS SCHEME

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21MAT41

Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024

Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. State and prove Cauchy's Riemann in polar form. **(06 Marks)**

b. Determine the analytic function f(z) = u + iv given that the real part $u = e^{2x}(x\cos 2y - y\sin 2y)$. **(07 Marks)**

c. Evaluate : $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the parabola $x = 2t$, $y = t^2 + 3$. **(07 Marks)**

OR

2 a. State and prove Cauchy's integral theorem. **(06 Marks)**

b. Evaluate $\int_C \frac{e^{2z}}{(z+1)(z-2)} dz$ where $C : |z| = 3$. **(07 Marks)**

c. If f(z) is analytic show that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4|f'(z)|^2$. **(07 Marks)**

Module-2

3 a. Obtain the series solution of Bessel's differential equation :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0.$$ **(06 Marks)**

b. If α and β are roots $J_n(x) = 8$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$. **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = a P_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$ find the values of a, b, c, d. **(07 Marks)**

OR

4 a. Prove that $P_n(x) = \frac{1}{2^n \cdot n!} \frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$. **(06 Marks)**

b. Prove that $P_3(\cos\theta) = \frac{1}{8}(3\cos\theta + 5\cos 3\theta)$. **(07 Marks)**

c. Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. **(07 Marks)**

Module-3

5 a. Find the coefficient of correlation and obtain the lines of regression for the following data :

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

(06 Marks)

b. The equations of regression lines of two variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$, find the correlation coefficient and means of x and y. (07 Marks)

c. Fit a curve of the form $y = a + bx$ for the following data hence find y at $x = 15$.

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(07 Marks)

OR

6 a. If the variable x and y such that :
i) $x + y$ has variance 15
ii) $x - y$ has variance 11
iii) $2x + y$ has variance 29 find σ_x, σ_y and coefficient of correlation. (06 Marks)

b. Fit a parabola $y = a + bx + cx^2$ to the following data :

x	1	2	3	4	5	6	7
y	2.3	5.2	9.7	16.5	9.4	35.5	54.4

(07 Marks)

c. Fit a curve of the form $y = ax^b$ for the following data :

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(07 Marks)

Module-4

7 a. The p.d.f of a variate x is given by the following data :

x	–2	–1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

Find the value of K. Also find $P(x \geq 0)$ and $P(-2 < x < 2)$. (06 Marks)

b. Derive the mean and variance of the Binomial distribution. (07 Marks)

c. If the mean and standard deviation of the number of correctly answered questions in a test given to 4096 students are 2.5 and $\sqrt{1.875}$. Find an estimate of the number of conditions answering correctly i) 8 or more questions ii) 2 or less. (07 Marks)

OR

8 a. The number of accidents in a year to taxi drivers in city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of the drivers with :
i) No accident in a year
ii) More than e accident in a year. (06 Marks)

b. Find the value of C such that $f(x) = \begin{cases} \frac{x}{6} + c & 0 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$ is p.d.f. Also find $P(1 \leq x \leq 2)$.

(07 Marks)

c. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation of the distribution. (07 Marks)

Module-5

9 a. x and y are independent random variable, x takes values 2, 5, 7 with the probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ respectively. y takes the values 3, 4, 5 with probability $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$.
i) Find the joint probability of X and Y
ii) Show that the covariance of X and Y is equal to zero. **(06 Marks)**

b. Define :
i) Null hypothesis
ii) Type – I and Type – II errors
iii) Degree of freedom
iv) Level of Significance. **(07 Marks)**

c. 4 coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness and fit. $(\chi^2_{0.05} = 9.49$ for 4 pd.f.). **(07 Marks)**

OR

10 a. In a hospital 230 females and 270 males were born in a year. Do these figures confirm the hypothesis that sexes are born in equal proportions. **(10 Marks)**

b. Random sample of 1000 engineering students from a city A and 800 form city B were taken. It was found that 400 students in each of the sample were from payment quota. Does the data reveal a significant different between the two cities in respect to payment quota students? **(10 Marks)**

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