17MATDIP31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024 **Additional Mathematics – I**

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

a. Define dot product between two vectors \vec{a} and \vec{b} . Find the sine of the angle between 1 $\vec{a} = 4\vec{i} - \vec{j} + 3\vec{k}$ and $\vec{b} = -2\vec{i} + \vec{j} - 2\vec{k}$. (08 Marks)

b. Express $\frac{3+4i}{3-4i}$ in the form of x+iy and hence find its modulus and amplitude. (06 Marks)

c. Find the real part of $\frac{1}{1+\cos\theta+i\sin\theta}$

(06 Marks)

a. Prove that $\left[\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right]^4 = \cos 8\theta + i\sin 8\theta$ (08 Marks)

b. If $\vec{A} = 2\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{B} = 2\vec{i} - 2\vec{j} + \vec{k}$. Show that $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are orthogonal.

(06 Marks)

c. Find the value of λ so that the vectors $\vec{A} = 3\vec{i} + 5\vec{j} - 3\vec{k}$, $\vec{B} = \vec{i} + \lambda \vec{j} + 2\vec{k}$ and $\vec{C} = 2\vec{i} - 2\vec{j} + \vec{k}$ are co-planar. (06 Marks)

a. If $y = tan^{-1}x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. b. Find the angle between the curves $r = a(1+cos\theta)$ and $r = b(1-cos\theta)$. (08 Marks)

(06 Marks)

c. If $u = \tan^{-1} \left[\frac{x^3 + y^3}{x - y} \right]$. Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. (06 Marks)

a. Obtain the Maclaurin's series expansion of tanx upto the term containing x⁵. (08 Marks)

b. Find the Pedal equation to the curve $r^m = a^m \cos m\theta$. (06 Marks)

c. If u = f(x - y, y - z, z - x), show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (06 Marks)

Module-3

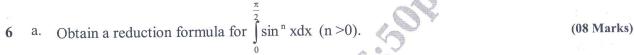
Obtain a reduction formula for $\int \cos^n x dx$ (n > 0). (08 Marks)

Evaluate $\int x \sqrt{ax - x^2} dx$. (06 Marks)

Evaluate $\iint xy dx dy$ where R is the I quadrant of the circle $x^2 + y^2 = a^2$, $x \ge 0$, $y \ge 0$.

(06 Marks)

OR



- b. Using reduction formula evaluate $\int_{0}^{1} x^{5} (1-x^{2})^{\frac{5}{2}} dx$. (06 Marks)
- c. Evaluate $\int_{0.01}^{1.22} x^2 yz dx dy dz$. (06 Marks)

Module-4

- 7 a. A particle moves along the curve $x = (1-t^3)$, $y = (1+t^2)$ and z = (2t-5). Determine the components of velocity and acceleration at t = 1 in the direction 2i + j + 2k. (08 Marks)
 - b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of the vector 2i j 2k. (06 Marks)
 - c. If $\vec{F} = (x+y+1)\vec{i} + \vec{j} (x+y)\vec{k}$, show that $\vec{F} \cdot \text{curl } \vec{F} = 0$ (06 Marks)

OR

8 a. Find div
$$\vec{F}$$
 and curl \vec{F} where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ (08 Marks)

b. Show that
$$\overrightarrow{F} = \frac{xi + yj}{x^2 + y^2}$$
 is solenoidal. (06 Marks)

c. Find the values of the constants a, b, c such that

$$\overrightarrow{F} = (x + y + az)i + (6x + 2y - z)j + (x + cy + 2z)k \text{ is irrotational.}$$
 (06 Marks)

Module-5

9 a. Solve:
$$(1+y^2)dx = (\tan^{-1} y - x) dy$$
. (08 Marks)

b. Solve:
$$(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$$
. (06 Marks)

c. Solve:
$$\left[x \tan\left(\frac{y}{x}\right) - y \sec^2\left(\frac{y}{x}\right)\right] dx + x \sec^2\left(\frac{y}{x}\right) dy = 0$$
. (06 Marks)

OR

10 a. Solve:
$$\frac{dy}{dx} + \frac{y}{x} = y^2 x$$
. (08 Marks)

b. Solve:
$$y(x+y)dx + (x+2y-1)dy = 0$$
. (06 Marks)

c. Solve:
$$\left[y^2 e^{xy^2} + 4x^3 \right] dx + \left[2xy e^{xy^2} - 3y^2 \right] dy = 0$$
 (06 Marks)

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