

CBCS SCHEME

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18MATDIP31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024

Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- Express $\sqrt{8} + 4i$ in the polar form and hence find its modulus and amplitude. (08 Marks)
 - Find the real part of $\frac{1}{1 + \cos\theta - i \sin\theta}$ (06 Marks)
 - Show that $(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i \sin\theta)^n = 2^{n+1} \cos^n\left(\frac{\theta}{2}\right) \cos\left(\frac{n\theta}{2}\right)$ (06 Marks)

OR

- If $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{B} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{C} = 3\hat{i} + \hat{j}$, find p such that $\vec{A} + p\vec{B}$ is perpendicular to \vec{C} . (08 Marks)
 - Find the area of the parallelogram whose adjacent sides are the vectors $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$. (06 Marks)
 - If $\vec{A} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{B} = 3\hat{i} - \hat{j} + 2\hat{k}$ then show that $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are orthogonal. (06 Marks)

Module-2

- Obtain the Maclaurin's series expansion of $\log(\sec x)$ upto the term containing x^3 . (08 Marks)
 - Using Euler's theorem, prove that $xu_x + yu_y = \frac{5}{2}u$ where $u = \frac{x^3 + y^3}{\sqrt{x+y}}$ (06 Marks)
 - If $u = f(x-y, y-z, z-x)$, then show that $u_x + u_y + u_z = 0$. (06 Marks)

OR

- Prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \dots$ by using Maclaurin's series. (08 Marks)
 - If $u = \sin^{-1}\left\{\frac{x^2 y^2}{x+y}\right\}$, then show that $xu_x + yu_y = 3 \tan u$, by using Euler's theorem. (06 Marks)
 - If $u = 2xy$, $v = x^2 - y^2$ and $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial(u, v)}{\partial(r, \theta)}$. (06 Marks)

Module-3

- A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$, $z = 2t - 5$ where t is time. Find the components of velocity and acceleration at $t = 1$ in the direction $2\hat{i} + \hat{j} + 2\hat{k}$ (08 Marks)
 - Find the unit normal to the surface $xy^3z^2 = 4$ at $(-1, -1, 2)$ (06 Marks)
 - Show that $\vec{F} = (x + y + z)\hat{i} + (x + 2y - z)\hat{j} + (x - y + 2z)\hat{k}$ is irrotational. (06 Marks)

OR

- 6 a. Find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ (08 Marks)
 b. If $\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$, then show that $\vec{F} \cdot \text{curl } \vec{F} = 0$. (06 Marks)
 c. Find the value of a such that $\vec{F} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solenoidal. (06 Marks)

Module-4

- 7 a. Evaluate

$$\int_0^{\pi/2} \sin^5 x \, dx$$

(08 Marks)

- b. Evaluate

$$\int_0^{\infty} \frac{x^4}{(1+x^2)^4} \, dx$$

(06 Marks)

- c. Evaluate

$$\iint_R (x^2 + y^2) \, dx \, dy \quad \text{where } R \text{ is the region bounded by } y = x \text{ and } y = x^2. \quad (06 \text{ Marks})$$

OR

- 8 a. Evaluate

$$\int_0^{\pi/2} \cos^6 x \, dx$$

(08 Marks)

- b. Evaluate

$$\int_0^a x \sqrt{ax - x^2} \, dx$$

(06 Marks)

- c. Evaluate

$$\int_0^a \int_0^b \int_0^c (x+y+z) \, dx \, dy \, dz$$

(06 Marks)

Module-5

- 9 a. Solve :
- $y(2x - y + 1)dx + x(3x - 4y + 3)dy = 0$
- (08 Marks)

b. Solve : $\frac{dx}{dy} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$ (06 Marks)

c. Solve : $\frac{dx}{dy} + \frac{2y}{x} = y^2 x$ (06 Marks)

OR

10 a. Solve : $\frac{dy}{dx} + \frac{y}{x} = y^2 x$ (08 Marks)

b. Solve : $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ (06 Marks)

c. Solve : $\frac{dy}{dx} + y \cot x = \cos x$ (06 Marks)
