USN | | | | | | | | | | 18MATDIP41

Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Additional Mathematics – II

Time: 3 hrs.　　　　　　　　　　　　　　　　　　　　　　Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1. a. Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. **(06 Marks)**

   b. Solve by using Gauss elimination method. Given $x + y + z = 9$, $2x + y - z = 0$ and $2x + 5y + 7z = 52$. **(07 Marks)**

   c. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **(07 Marks)**

OR

2. a. Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. **(06 Marks)**

   b. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(07 Marks)**

   c. Find the values of λ and μ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(07 Marks)**

Module-2

3. a. Using Newton Raphson method, find the real root of the equation $3x = \cos x + 1$, correct to four decimal places. Take $x = 0.6$ as the initial approximation. **(06 Marks)**

   b. Given $f(40) = 184$, $f(50) = 204$, $f(60) = 226$, $f(70) = 250$, $f(80) = 276$, $f(90) = 304$. Find $f(85)$ using Newton's backward difference interpolation formula. **(07 Marks)**

   c. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Simpson's $\frac{1}{3}$ rule by considering 6 subintervals. **(07 Marks)**

OR

4. a. Using Regula Falsi method, find a real root of the equation $x \log_{10} x - 1.2 = 0$ which lies in (2, 3). Carryout 3 iterations. **(06 Marks)**

   b. Using the following data, find y when x = 1. Given,

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

   Use Newton's forward interpolation formula. **(07 Marks)**

   c. Evaluate $\int_4^{5.2} \log x \, dx$ by using Weddle's rules taking 6 subintervals. **(07 Marks)**

Module-3

5   a.   Solve $(D^3 + 3D^2 + 3D + 1)y = 0$.   **(06 Marks)**

b.   Solve $(D^2 + 7D + 12)y = \cosh x$.   **(07 Marks)**

c.   Solve $(D^2 - 4D + 4)y = \cos 2x$.   **(07 Marks)**

OR

6   a.   Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$.   **(06 Marks)**

b.   Solve $(D^2 - 6D + 9)y = 6e^{3x}$.   **(07 Marks)**

c.   Solve $(D^2 - 5D + 6)y = \sin 3x$.   **(07 Marks)**

Module-4

7   a.   Form the partial differential equation by eliminating arbitrary functions from $z = y^2 + 2f\left(\dfrac{1}{x} + \log y\right)$.   **(06 Marks)**

b.   Form the PDE by eliminating arbitrary constants a and b from the relation $(x - a)^2 + (y - b)^2 + z^2 = k^2$.   **(07 Marks)**

c.   Solve $\dfrac{\partial^2 z}{\partial x^2} = a^2 z$, given that when x = 0, z = 0 and $\dfrac{\partial z}{\partial x} = a \sin y$.   **(07 Marks)**

OR

8   a.   Form a partial differential equation by eliminating the arbitrary function from $\phi(x + y + z,\ x^2 + y^2 + z^2) = 0$.   **(06 Marks)**

b.   Form a partial differential equation by eliminating arbitrary function from $z = f(x + ct) + g(x - ct)$.   **(07 Marks)**

c.   Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ by direct integration. Given that $\dfrac{\partial z}{\partial y} = -2 \sin y$ when x = 0 and z = 0 when y is an odd multiple of $\dfrac{\pi}{2}$.   **(07 Marks)**

Module-5

9   a.   Given $P(A) = \dfrac{1}{4}$, $P(B) = \dfrac{1}{3}$ and $P(A \cup B) = \dfrac{1}{2}$. Find $P(A/B)$, $P(B/A)$, $P(A \cap \overline{B})$ and $P(A/\overline{B})$.   **(06 Marks)**

b.   The probability that three students A, B, C, solve a problem is $\dfrac{1}{2}$, $\dfrac{1}{3}$, $\dfrac{1}{4}$ respectively. If the problem is simultaneously assigned to all of them, what is the probability that the problem is solved?   **(07 Marks)**

c.   State and prove Baye's theorem.   **(07 Marks)**

OR

10   a.   If A and B are independent events, show that $\overline{A}$ and $\overline{B}$ are also independent.   **(06 Marks)**

b.   The probability that a team wins a match is $\dfrac{3}{5}$. If this team plays 3 matches in a tournament, what is the probability that the team wins (i) atleast one match (ii) all matches.   **(07 Marks)**

c.   An office has 4 secretaries handling respectively 20%, 60% and 15% and 5% of the files of all government reports. The probability that they misfile such reports is respectively 0.05, 0.1 and 0.05. Find the probability that a misfiled report can be blamed on first secretary?   **(07 Marks)**

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