



## Third Semester B.E. Degree Examination, June/July 2024 Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. Obtain the fourier series of the function  $f(x) = x - x^2$  in  $-\pi \leq x \leq \pi$  and hence deduce  $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  (08 Marks)
- b. Obtain the Half Range Fourier cosine series for the  $f(x) = \sin x$  in  $[0, \pi]$ . (06 Marks)
- c. Obtain the constant term and the coefficients of first sine and cosine terms in the fourier expansion of  $y$  given

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

(06 Marks)

**OR**

- 2 a. Obtain the fourier series of  $f(x) = \frac{\pi - x}{2}$  in  $[0, 2\pi]$  and hence deduce that  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$  (08 Marks)
- b. Find the fourier half range cosine series of the function  $f(x) = 2x - x^2$  in  $[0, 3]$  (06 Marks)
- c. Express  $y$  as a fourier series upto first harmonic given

x :	0	30	60	90	120	150	180	210	240	270	300	330
y :	1.8	1.1	0.30	0.16	1.5	1.3	2.16	1.25	1.3	1.52	1.76	2.0

(06 Marks)

### Module-2

- 3 a. Find the complex Fourier transform of the function :  $f(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$  and hence evaluate  $\int_0^{\infty} \frac{\sin x}{x} dx$ . (08 Marks)
- b. Find the Fourier cosine transform of  $e^{-ax}$ . (06 Marks)
- c. Solve by using  $z$  - transforms  $u_{n+2} - 4u_n = 0$  given that  $u_0 = 0$  and  $u_1 = 2$ . (06 Marks)

**OR**

- 4 a. Find the Fourier sine and Cosine transforms of :  $f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$  (08 Marks)
- b. Find the  $Z$  - transform of : i)  $n^2$  ii)  $ne^{-an}$ . (06 Marks)
- c. Obtain the inverse  $Z$  - transform of  $\frac{2z^2 + 3z}{(z+2)(z-4)}$ . (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

**Module-3**

- 5 a. Find the correlation coefficient using the following table as values: (08 Marks)
- |   |    |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|
| x | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |
- b. Obtain an equation of the form  $y = ax + b$  given that, (06 Marks)
- |   |    |    |    |    |    |    |
|---|----|----|----|----|----|----|
| x | 0  | 5  | 10 | 15 | 20 | 25 |
| y | 12 | 15 | 17 | 22 | 24 | 30 |
- c. Apply Regula-Falsi method to find the root of  $xe^x = \cos x$  in four approximations with four decimals in (0, 1). (06 Marks)

**OR**

- 6 a. Obtain the regression line of y on x for the following table of values: (08 Marks)
- |   |   |   |    |    |    |    |    |    |    |
|---|---|---|----|----|----|----|----|----|----|
| x | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 | 16 | 15 |
- b. Fit a parabola  $y = a + bx + cx^2$  to the following data: (06 Marks)
- |   |     |     |      |      |      |     |
|---|-----|-----|------|------|------|-----|
| x | 20  | 40  | 60   | 80   | 100  | 120 |
| y | 5.5 | 9.1 | 14.9 | 22.8 | 33.3 | 46  |
- c. Find the root of the equation  $x^4 - x - 9 = 0$  by Newton-Raphson method in three approximations with three decimal places. (Take  $x_0 = 2$ ) (06 Marks)

**Module-4**

- 7 a. From the following table, estimate the number of students who obtained marks between 40 and 45.

Marks :	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
No. of students	31	42	51	35	31

(08 Marks)

- b. Use Newton's dividend formula to find  $f(9)$  for the data:

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

(06 Marks)

- c. Find the approximate value of  $\int_0^{\pi/2} \sqrt{\cos \theta} d\theta$  by Simpson's  $\frac{1}{3}$ rd rule by dividing  $\left[0, \frac{\pi}{2}\right]$  into 6 equal parts. (06 Marks)

**OR**

- 8 a. The area A of a circle of diameter d is given for the following values:

d :	80	85	90	95	100
a :	5026	5674	6362	7088	7854

Calculate the area of circle of diameter 105 by Newton's backward formula. (08 Marks)

- b. Using Lagrange's interpolation formula to find the polynomial which passes through the points (0, -12), (1, 0), (3, 6), (4, 12). (06 Marks)

- c. Evaluate  $\int_4^{5.2} \log_e x dx$  taking 6 equal parts by applying Weddle's rule. (06 Marks)

**Module-5**

- 9 a. Verify Green's theorem in the plane for  $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ , where  $C$  is the boundary of the region defined by  $x = 0$ ,  $y = 0$ ,  $x + y = 1$ . (08 Marks)
- b. Evaluate  $\oint_C \vec{F} \cdot d\vec{r}$  by Stoke's theorem with  $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$  and  $C$  is the boundary of the triangle with vertices at,  $(0, 0, 0)$ ,  $(1, 0, 0)$  and  $(1, 1, 0)$ . (06 Marks)
- c. Show that the geodesies on a plane are straight lines. (06 Marks)

**OR**

- 10 a. Find  $\iint_S \vec{F} \cdot d\vec{S}$ , where  $F = (2x + 3z)\hat{i} - (xz + y)\hat{j} + (y^2 + 2z)\hat{k}$  and  $S$  is the surface of the sphere having center at  $(3, -1, 2)$  and radius 3. (Use Gauss divergence theorem). (08 Marks)
- b. Derive Euler's equation with usual notations as,  $\frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0$ . (06 Marks)
- c. Find the extremals of the functional,  

$$\int_{x_0}^{x_1} \left( \frac{y'^2}{x^3} \right) dx.$$
 (06 Marks)

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