CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐

BMATE301/BEE301

Third Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024

Mathematics-III for EE Engineering

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.
4. Mathematics handbook is permitted.

<table>
<tr><th colspan="3">Module – 1</th><th>M</th><th>L</th><th>C</th></tr>
<tr><td>Q.1</td><td>a.</td><td>Solve : $(D^4 + 8D^2 + 16)y = 0$.</td><td>6</td><td>L1</td><td>CO1</td></tr>
<tr><td></td><td>b.</td><td>Solve : $(D^3 - 3D + 2)y = 2\sinh x$</td><td>7</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>c.</td><td>Solve : $x^2y'' - 3xy' + 5y = 3\sin(\log x)$</td><td>7</td><td>L3</td><td>CO1</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.2</td><td>a.</td><td>Solve : $(D^4 - 4D^3 - 5D^2 - 36D - 36)y = 0$.</td><td>6</td><td>L1</td><td>CO1</td></tr>
<tr><td></td><td>b.</td><td>Solve : $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = \sin 2x$.</td><td>7</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>c.</td><td>Solve : $(2x+1)^2\frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 3(2x+1)$.</td><td>7</td><td>L3</td><td>CO1</td></tr>
<tr><td colspan="6">Module – 2</td></tr>
<tr><td>Q.3</td><td>a.</td><td>Find the curve at best fit of the form $y = ax^6$ to the following data :
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y</td><td>0.5</td><td>2</td><td>4.5</td><td>8</td><td>12.5</td></tr>
</table></td><td>6</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>b.</td><td>Calculate the coefficient of correlation and obtain the lines of regression for the following data :
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>y</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td><td>16</td><td>15</td></tr>
</table></td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>In a partially destroyed laboratory record of correlation data, following results only available : Variance of x is 9 and regression lines, $4x - 5y + 33 = 0$; $20x - 9y = 107$. Find
(i) Mean value of x and y
(ii) SD of y.
(iii) Coefficient of correlation between x and y.</td><td>7</td><td>L4</td><td>CO2</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.4</td><td>a.</td><td>Fit a curve of the form, $y = ax^2 + bx + c$ to the following data :
<table>
<tr><td>x :</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y :</td><td>10</td><td>12</td><td>8</td><td>10</td><td>14</td></tr>
</table></td><td>6</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>b.</td><td>If θ is the acute angle between the two regression lines relating the variables x and y, show that $\tan\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}$.
Indicate the significance of the cases $r = 0$ and $r = \pm 1$</td><td>7</td><td>L2</td><td>CO2</td></tr>
</table>

<table>
<tr><td></td><td>c.</td><td>Ten competitor's in a music contest ranked by 3 judges A, B, C in the following order. Use the rank correlation coefficient to decide which pair judges have the nearest approach to common test of music.
<table>
<tr><td>A</td><td>1</td><td>6</td><td>5</td><td>10</td><td>3</td><td>2</td><td>4</td><td>9</td><td>7</td><td>8</td></tr>
<tr><td>B</td><td>3</td><td>5</td><td>8</td><td>4</td><td>7</td><td>10</td><td>2</td><td>1</td><td>6</td><td>9</td></tr>
<tr><td>C</td><td>6</td><td>4</td><td>9</td><td>8</td><td>1</td><td>2</td><td>3</td><td>10</td><td>5</td><td>7</td></tr>
</table></td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td colspan="6">Module – 3</td></tr>
<tr><td>Q.5</td><td>a.</td><td>Find the Fourier series for the function $f(x)=x^2$ in the interval $-\pi \le x \le \pi$, hence deduce the $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Expand the function $f(x)=x(\pi-x)$ over the interval $(0,\pi)$ in half range cosine Fourier series hence deduce that $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}=\frac{\pi^2}{12}$</td><td>7</td><td>L3</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>The following table gives the variations of a periodic current A over a certain period T.
<table>
<tr><td>t(sec)</td><td>0</td><td>$\frac{T}{6}$</td><td>$\frac{T}{3}$</td><td>$\frac{T}{2}$</td><td>$\frac{2T}{3}$</td><td>$\frac{5T}{6}$</td><td>T</td></tr>
<tr><td>A(amp)</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr>
</table>
Show that there is a current part of 0.75 amp in the current A and obtain the amplitude of the first harmonic.</td><td>7</td><td>L3</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.6</td><td>a.</td><td>Find the Fourier expansion of the function $f(x)=(\pi-x)^2$ over the interval $0 \le x \le 2\pi$. Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Expand the function $f(x)=\begin{cases}\frac{1}{4}-x & \text{for } 0<x<\frac{1}{2}\\ 0-\frac{3}{4} & \frac{1}{2}<x<1\end{cases}$ in the half range sine series.</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Find the constant term and the first harmonic in the Fourier series for $f(x)$ given by the table.
<table>
<tr><td>x</td><td>0</td><td>$\frac{\pi}{3}$</td><td>$\frac{2\pi}{3}$</td><td>π</td><td>$\frac{4\pi}{3}$</td><td>$\frac{5\pi}{3}$</td><td>2π</td></tr>
<tr><td>f(x)</td><td>1.0</td><td>1.4</td><td>1.9</td><td>1.7</td><td>1.5</td><td>1.2</td><td>1.0</td></tr>
</table></td><td>7</td><td>L3</td><td>CO3</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>Q.7</td><td>a.</td><td>Find the Fourier transform of the function, $f(x)=\begin{cases}1 & \text{for } |x|\le a\\ 0 & \text{for } |x|\ge a\end{cases}$
where a is a positive constant hence evaluate integrals ,
$\int_{-\infty}^{\infty}\frac{\sin ax\cos ax}{x}dx$</td><td>6</td><td>L2</td><td>CO4</td></tr>
</table>

<table>
<tr><td></td><td>b.</td><td>Find the Fourier cosine transform of $f(x)=e^{-ax}$, a > 0, hence deduce that $\int_0^\infty \frac{\cos mx}{x^2+a^2}dx=\frac{\pi}{2a}e^{-am}$</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Find the inverse z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6" align="center">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>Find the Fourier transform of $f(x)=e^{-\frac{x^2}{2}}$.</td><td>6</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Find the z-transform of $\sin n\theta$ and $\cos n\theta$ hence find $z\left\{\cos\left(\frac{n\pi}{2}\right)\right\}$ and $z\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Solve the difference equation, $u_{n+2}-5u_{n+1}+6u_n=2$ given $u_0=3$, $u_1=7$, using z-transforms.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6" align="center">Module – 5</td></tr>
<tr><td>Q.9</td><td>a.</td><td>Define (i) Type I and Type II errors.
(ii) Confidence interval.
(iii) Level of significance.</td><td>6</td><td>L1</td><td>CO5</td></tr>
<tr><td></td><td>b.</td><td>The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that
(i) Exactly 2 will be defective.
(ii) At least 2 will be defective.
(iii) None will be defective</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>In normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and SD, given that A(0.5) = 0.19 and A(1.4) = 0.42, where A(Z) is the area under the standard normal curve from 0 to z.</td><td>7</td><td>L3</td><td>CO5</td></tr>
<tr><td colspan="6" align="center">OR</td></tr>
<tr><td>Q.10</td><td>a.</td><td>The pdf P(x) of a variate X is given by the table :
<table>
<tr><td>x :</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>P(x) :</td><td>K</td><td>3K</td><td>5K</td><td>7K</td><td>9K</td><td>11K</td><td>13K</td></tr>
</table>
For what value of K, does this represent a valid probability distribution? Also find $P(x<4)$, $P(x\ge 5)$ and $P(3<x\le 6)$.</td><td>6</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b.</td><td>Consider the sample consisting of nine numbers, 45, 47, 50, 52, 48, 47, 49, 53 and 51. The sample is drawn from a population whose mean is 47.5. Find whether the sample mean differs significantly from the population mean at 5% level of significance (Given $t_{0.05}(df=8)=2.31$)</td><td>7</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given by the table :
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>frequency</td><td>15</td><td>6</td><td>4</td><td>7</td><td>11</td><td>17</td></tr>
</table>
Test the hypothesis that the die is unbiased. Given $\chi^2_{0.05}(5)=11.07$ and $\chi^2_{0.01}(5)=15.09$.</td><td>7</td><td>L3</td><td>CO5</td></tr>
</table>

* * * * *