CBCS SCHEME

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18MAT41

Fourth Semester B.E. Degree Examination, June/July 2024
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive Cauchy - Riemann equations in Cartesian form. **(06 Marks)**

b. Show that the function $v = (\sin x \cosh y + 2 \text{ los } x \ \sinh y) + (x^2 - y^2 + 4xy)$ is harmonic and hence find Analytic function. **(07 Marks)**

c. Verify that $v = \frac{1}{r^2}(\cos 2\theta)$, $r \neq 0$ is harmonic. Find an analytic function f(z) whose real part is u. **(07 Marks)**

OR

2 a. Derive Cauchy-Riemann equations in polar form. **(06 Marks)**

b. Given $f(z) = u + iv$ an analytic function and prove the following property:

$$\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$$ **(07 Marks)**

c. Find an analytic function $f(z) = u + iv$, given
$u - v = e^x(\cos y - \sin y)$ **(07 Marks)**

Module-2

3 a. Discuss the transformation $w = e^z$. Show the transform in z-plane and w-plane. **(06 Marks)**

b. Evaluate $\int_c \frac{e^z}{(z-2)(z-5)^3} dz$, where c is the circle $|z| = 8$. **(07 Marks)**

c. Evaluate $\int_{z=0}^{z=1+i} (x^2 - iy)dz$ along the following curves:

i) The straight line $y = x$ ii) The parabola $y = x^2$. **(07 Marks)**

OR

4 a. Find the bilinear transformation that maps the points z = -1, i, 1 onto the points w = 1, i, -1 respectively. **(06 Marks)**

b. Discuss the transformation $w = z + \frac{1}{z}$. Show the transform in z and w planes. **(07 Marks)**

c. State and prove Cauchy's integral formula. **(07 Marks)**

Module-3

5 a. Find the value of k such that the following table represents a finite probability distribution:

x :	-3	-2	-1	0	1	2	3
$P(x_i)$:	k	2k	3k	4k	3k	2k	k

Find the mean and the standard deviation of the distribution Also find $P(x > 1)$ and $P(-1 < x \leq 2)$. **(06 Marks)**

b. In a certain factory turning out razor blades, there is a small chance of 0.002, for a blade to be defective. The blades are supplied in packets of 10. Using Poisson distribution, calculate the approximate number of packets containing i) no defective ii) one defective iii) two defective blades in a consignment of 10,000 packets. **(07 Marks)**

c. For the normal distribution with mean 2 and standard deviation 4, calculate the following probabilities:
i) $P(x \geq 5)$ ii) $P\{|x| < 4\}$ iii) $P\{|x| > 3\}$ **(07 Marks)**

OR

6 a. A fair coin is tossed three times. Let x denotes the number of heads showing up. Find the distribution of x. Also find its mean variance and standard deviation. **(06 Marks)**

b. An underground mine has 5 pumps installed for pumping out storm water, the probability of any of the pumps failing during the storm is 1/8. What is the probability that
i) At least 2 pumps will be working ii) All pumps will be working during a particular storm? **(07 Marks)**

c. At a certain city bus stop, three buses arrive per hour on an average. Assuming that the time between successive arrivals is exponentially distributed, find the probability that the time between the arrivals of successive buses is
i) less than 10 minutes ii) at least 30 minutes. **(07 Marks)**

Module-4

7 a. If F is the force required to lift a load W, by mass of a pulley, fit a linear expression $F = a + bW$ against the following data:

W	50	70	100	120
F	12	15	21	25

(07 Marks)

b. Employ the formula $r = \dfrac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$ to determine the coefficient of correlation r, for the following data:

x :	92	89	87	86	83	77	71	63	53	50
y :	86	83	91	77	68	85	52	82	37	57

(07 Marks)

c. The tangent of the angle θ between the lines of regression of y on x and x on y is 0.6 and the standard deviation of y is twice the standard deviation of x, find the coefficient of correlation between x and y. **(06 Marks)**

OR

8 a. Fit a second-degree parabola in the form $y = a + bx + cx^2$ for the following data:

x :	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y :	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(06 Marks)

b. Obtain the lines of regression and hence find the coefficient of correlation for the following data:

x :	1	3	4	2	5	8	9	10	13	15
y :	8	6	10	8	12	16	16	10	32	32

(07 Marks)

c. Fit a curve of best fit of the form $y = ax^b$ to the following data:

x :	1	2	3	4	5
y :	0.5	2	4.5	8	12.5

(07 Marks)

Module-5

9 a. The joint probability function for two discrete random variables X and Y is given by $f(x, y) = c(2x + y)$ where x and y can assume all integral values such that $0 \le x \le 2$ and $0 \le y \le 3$ and $f(x, y) = 0$ otherwise.
Find i) The value of constant c ii) $P(X = 2, Y = 1)$ iii) $P(X \ge 1, Y \le 2)$ iv) $P[(x + y) \le 1]$ **(10 Marks)**

b. Define Type-I and Type-II errors. A coin was tossed 400 times and returned heads 216 times. Test the hypothesis that the coin is unbiased. **(10 Marks)**

OR

10 a. The life time of electric bulbs for a random sampling of 10from a large shipment gave the following data:

Item	1	2	3	4	5	6	7	8	9	10
Life in ‘1000s of hrs	4.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6

Can we accept the hypothesis that the average life time of bulbs is 4000 hrs. **(10 Marks)**

b. A joint distribution is given by the following table:

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find the marginal distribution of X and Y evaluate μ_X, μ_Y, σ_X, σ_Y. **(10 Marks)**

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