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21MAT41

Fourth Semester B.E. Degree Examination, June/July 2024 Complex Analysis, Probability and Statistical Methods

Time: 3 hrs.

Max. Marks: 100

- Note:1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of Normal distribution tables can be permitted.
3. Use of Students distribution tables can be permitted.*

Module-1

- 1 a. Derive the C-R equations in Polar form. (06 Marks)
- b. Construct the analytic function whose real part is $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$. (07 Marks)
- c. If $f(z)$ is a regular function, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (07 Marks)

OR

- 2 a. State and prove the Cauchy's integral formula. (06 Marks)
- b. Show that $f(z) = e^x (\cos y + i \sin y)$ is analytic and find its derivative. (07 Marks)
- c. Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$, where $C: |z|=3$. (07 Marks)

Module-2

- 3 a. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. (06 Marks)
- b. Prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$, $\alpha \neq \beta$. (07 Marks)
- c. Express $x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomial. (07 Marks)

OR

- 4 a. Show that $J_{-n}(x) = (-1)^n J_n(x)$. (06 Marks)
- b. Prove that $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$. (07 Marks)
- c. Show that $x^3 - 5x^2 + x + 2 = \frac{2}{5}P_3(x) - \frac{10}{3}P_2(x) + \frac{8}{5}P_1(x) + \frac{1}{6}P_0(x)$. (07 Marks)

Module-3

- 5 a. Find the regression line y on x and calculate y when $x = > 0$.

x:	71	68	66	67	70	71	70	73	72	65	66
y:	69	64	65	63	65	62	65	64	66	59	62

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

- b. Ten participants in a contest are ranked by two judges as follows:

x:	1	6	5	10	3	2	4	9	7	8
y:	6	4	9	8	1	2	3	10	5	7

Calculate the rank co-efficient of correlation.

(07 Marks)

- c. Fit a curve $y = ax^b$ from the following data :

x:	1	2	3	4	5
y:	0.5	2.0	4.5	8.0	12.5

(07 Marks)

OR

- 6 a. Given the equation of the lines $8x - 10y + 66 = 0$ and $40x - 18y = 214$. Compute the mean's of x and y , the coefficient of correlation and find σ_y if $\sigma_x = 3$.

(06 Marks)

- b. Fit a second degree parabola $y = ax^2 + bx + c$ in the least squares for the following data :

x:	1	2	3	4	5
y:	10	12	13	16	19

(07 Marks)

- c. Find the lines of regression of the following data :

x:	1	2	3	4	5	6	7
y:	9	8	10	12	11	13	14

(07 Marks)

Module-4

- 7 a. A random variable X has the following probability density function :

X:	-2	-1	0	1	2	3
P(X):	0.1	K	0.2	2K	0.3	K

Find the value of K , mean and variance.

(06 Marks)

- b. Derive the mean and variance of Binomial distribution.

(07 Marks)

- c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be,
(i) less than 65 (ii) more than 75 (iii) between 65 and 75.

(07 Marks)

OR

- 8 a. A random variable X has the pdf $f(x) = \begin{cases} Kx^2, & -3 < x < 3 \\ 0, & \text{elsewhere} \end{cases}$

Evaluate K , find (i) $P(1 \leq x \leq 2)$ (ii) $P(x \leq 2)$ (iii) $P(x > 1)$.

(06 Marks)

- b. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.

(07 Marks)

- c. In an examination 7% of students score less than 35 marks and 89% of students score less than 60 marks. Find the mean and standard deviation, if the marks are normally distributed.

(07 Marks)

Module-5

- 9 a. The following joint probability distribution of the random variable X and Y as follows:

	Y	1	3	9
X				
2		$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4		$\frac{1}{4}$	$\frac{1}{4}$	0
6		$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Determine the marginal distributions of X and Y . Find (i) $E(X)$ and $E(Y)$ (ii) $COV(X, Y)$.

(06 Marks)

- b. A sample of 900 items has mean 3.4 and S.D 2.61. Can the sample be regarded from population with mean 3.25 at 5% LOS? (07 Marks)
- c. The theory predicts the proportion be in the four groups G_1, G_2, G_3, G_4 should be in the ratio 9 : 3 : 3 : 1. In experiment with 1600 beans the numbers in the groups were 882, 313, 287 and 118. Do the experimental result support the theory. (07 Marks)

OR

- 10 a. Define the terms : (i) Type – I and Type – II errors (ii) Null hypothesis (iii) Level of significance. (06 Marks)
- b. A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with S.D. of 0.04 inch. On the basis of this sample would you say that the work is inferior? (07 Marks)
- c. Fit a Poisson distribution to the following data is and test for its goodness of fit at 5% LOS.

x	0	1	2	3	4
y	419	352	154	56	19

(07 Marks)
