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15MATDIP31

Third Semester B.E. Degree Examination, June/July 2024
Additional Mathematics – I

Time: 3 hrs.   Max. Marks : 80

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Express $\dfrac{3+4i}{3-4i}$ in the form $x + iy$. **(06 Marks)**

   b. Express $\sqrt{3}+i$ in the polar form and hence find their modulus and amplitudes. **(05 Marks)**

   c. Find the sine of the angle between $\vec{a} = 2i - 2j + k$ and $\vec{b} = i - 2j + 2k$. **(05 Marks)**

OR

2 a. Simplify :

$$\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta + i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 + (\cos 5\theta + i\sin 5\theta)^{-4}}.$$ **(06 Marks)**

   b. If $\vec{a} = i + 2j - 3k$ and $\vec{b} = 3i - j + 2k$, then show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are orthogonal. **(05 Marks)**

   c. Find the value of λ, so that the vectors $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = j + \lambda k$ are co-planar. **(05 Marks)**

Module-2

3 a. Obtain the nth derivative of $\sin(ax + b)$. **(05 Marks)**

   b. Find the pedal equation of $r^n = a^n \cos n\theta$. **(05 Marks)**

   c. If $u = \dfrac{yz}{x}$, $v = \dfrac{zx}{y}$, $w = \dfrac{xy}{z}$, show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. **(06 Marks)**

OR

4 a. If $u = \log\left(\dfrac{x^4 + y^4}{x + y}\right)$ show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3$. **(05 Marks)**

   b. If $u = f(x - y, y - z, z - x)$, show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(05 Marks)**

   c. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$ **(06 Marks)**

Module-3

5 a. Evaluate $\displaystyle\int_0^{\pi/6} \sin^6 3x \, dx$ using Reduction formula. **(06 Marks)**

   b. Evaluate $\displaystyle\int_0^1 x^6 \sqrt{1 - x^2} \, dx$ – using Reduction formula. **(05 Marks)**

   c. Evaluate $\displaystyle\int_1^2 \int_0^{2-y} xy\,dx\,dy$. **(05 Marks)**

OR

6 a. Evaluate $\int_{0}^{\pi/2} \sin^3 x \cos^7 x \, dx$. (06 Marks)

b. Evaluate $\int_{0}^{\pi} x \cos^6 x \, dx$. (05 Marks)

c. Evaluate $\int_{0}^{3} \int_{0}^{2} \int_{0}^{1} (x + y + z) \, dz \, dx \, dy$. (05 Marks)

Module-4

7 a. A particle moves along a curve $x = e^{-t}$, $y = 2\cos3t$, $z = 2\sin3t$, where t is the time variable. Determine the magnitudes of velocity and acceleration at t = 0. (05 Marks)

b. If $\phi = \log(x^2 + y^2 + z^2)$, find the magnitude of the grad ϕ at (1, 2, 3). (05 Marks)

c. If $\vec{R} = x\hat{i} + y\hat{i} + z\hat{k}$ find $\nabla \cdot \vec{R}$ and $\nabla \times \vec{R}$. (06 Marks)

OR

8 a. Find a unit vector normal to the surface $xy^3z^2 = 4$ at (-1, -1, 2). (05 Marks)

b. Find the value of 'a' if the vector $(ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal. (05 Marks)

c. Show that grad $(x^3 + y^3 + z^3 - 3xyz)$ is irrotational. (06 Marks)

Module-5

9 a. Solve $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$. (05 Marks)

b. Solve $(y^3 - 3x^2y)dx + (3xy^2 - x^3)dy = 0$. (05 Marks)

c. Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = xy^2$. (06 Marks)

OR

10 a. Solve $\dfrac{dy}{dx} + y \cot x = \cos x$. (05 Marks)

b. Solve $x^2y \, dx - (x^3 + y^3)dy = 0$ (05 Marks)

c. Solve $y(x + y)dx + (x + 2y - 1)dy = 0$ (06 Marks)

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