CBCS SCHEME

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BCS405A

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define a tautology. Prove that for any propositions p, q, r the compound propositions $\{(p \to q) \wedge (q \to r)\} \to (p \to r)$ is tautology.	06	L2	CO1
	b.	Establish the validity of the following argument using the rules of inference: $\{p \wedge (p \to q) \wedge (s \vee r) \wedge (r \to \sim q)\} \to (s \vee t)$	07	L2	CO1
	c.	For any two odd integers m and n, show that: (i) $m + n$ is even (ii) mn is odd	07	L2	CO1
		OR			
Q.2	a.	Show that the compound proposition $[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$ for primitive statements p, q, r is logically equivalent.	06	L2	CO1
	b.	Prove the following using law of logic: $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$	07	L2	CO1
	c.	Determine the truth value of each of the following quantified statements, the universe being the set of all non-zero integers: (i) $\exists x, \exists y, [xy = 1]$ (ii) $\exists x, \forall y, [xy = 1]$ (iii) $\forall x, \exists y, [xy = 1]$ (iv) $\exists x, \exists y, [(2x + y = 5) \wedge (x - 3y = -8)]$ (v) $\exists x, \exists y, [(3x - y = 17) \wedge (2x + 4y = 3)]$	07	L3	CO1
		Module – 2			
Q.3	a.	Prove that for each $n \in z^+$, $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$.	06	L2	CO2
	b.	Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$, prove that $a_n \leq 3^n \ \forall \ n \in z^+$.	07	L2	CO2
	c.	How many positive integers n can be we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?	07	L3	CO2
		OR			
Q.4	a.	By mathematical induction prove that $1.3 + 2.4 + \ldots. + n(n+2) = \frac{n(n+1)(2n+7)}{6}$	06	L2	CO2
	b.	Find the number of permutations of the letters of the word ENGINEERING such that: (i) All the E's are together (ii) Arrangement begin with N (iii) All the vowels are adjacent.	07	L3	CO2
	c.	Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$.	07	L3	CO2
		Module – 3			
Q.5	a.	State pigeon hole principle. Prove that if 30 dictionaries in a library contain a total of 61,327 pages then atleast one of the dictionaries must have atleast 2045 pages.	06	L3	CO3
	b.	Let $f : R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{if } x > 0 \\ 1 - 3x & \text{if } x \leq 0 \end{cases}$. Find $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(-1)$, $f^{-1}(3)$, $f^{-1}(-6)$, $f^{-1}([-6, 5])$ and $f^{-1}([-5, 5])$	07	L2	CO3
	c.	Draw the Hasse diagram representing the positive divisor of 36.	07	L3	CO3

OR

Q.6	a.	Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6}, (i) How many functions are there from A to B? (ii) How many of these are one to one? (iii) How many functions are there from B to A? (iv) How many of these are onto?	06	L2	CO3
	b.	Let f and g be functions from R to R defined by f(x) = ax + b and $g(x) = 1 - x + x^2$. If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a and b.	07	L2	CO3
	c.	Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aRb if and only if "a is multiple of b". Write down the relation R, relation matrix M(R) and draw the digraph. List out in degree and out degree.	07	L3	CO3
		Module – 4			
Q.7	a.	In how many ways 5 number of a's, 4 number of b's and 3 number of c's can be arranged so that all the identical letters are not in a single block?	06	L3	CO4
	b.	Determine the number of positive integers n such that $1 \le n \le 100$ and n is not divisible by 2, 3, or 5.	07	L3	CO4
	c.	Solve the recurrence relation $a_{n+2} - 3a_{n+1} + 2a_n = 0$, $a_0 = 1$, $a_1 = 6$.	07	L2	CO4
		OR			
Q.8	a.	In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs?	06	L3	CO4
	b.	Five teachers T_1, T_2, T_3, T_4 are to be made class teachers for five classes, C_1, C_2, C_3, C_4, C_5, one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5, and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned the work? (Without displeasing any teacher)	07	L3	CO4
	c.	Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ where $n \ge 0$ and $F_0 = 0$, $F_1 = 1$.	07	L2	CO4
		Module – 5			
Q.9	a.	If G be a set of all non zero real numbers and let $a * b = \frac{ab}{2}$ then show that (G, ∗) is an abelian group.	06	L2	CO5
	b.	Define Klein group and if A = {e, a, b, c} then show that this is a Klein-4 group.	07	L2	CO5
	c.	State and prove Lagrange's theorem.	07	L2	CO5
		OR			
Q.10	a.	If H and K are subgroups of group G, prove that $H \cap K$ is also a subgroup of G. Is $H \cup K$ a subgroup of G?	06	L2	CO5
	b.	Define cyclic group and show that (G, ∗) whose multiplication table is as given below is cyclic.	07	L2	CO5

∗	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e

	c.	Let $G = S_4$, for $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, find the subgroup $H = \langle \alpha \rangle$. Determine the left cosets of H in G.	07	L3	CO5

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