

Fourth Semester B.E. Degree Examination, Dec.2024/Jan.2025 Signals and Systems

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Check whether the following signals are periodic or not. If periodic find the fundamental period.
 - i) $x(t) = \sin\left(\frac{\pi}{3}t\right) + 2 \cos\left(\frac{8\pi}{3}t\right)$
 - ii) $x[n] = \sin\left(\frac{3\pi n}{4}\right) \sin\left(\frac{\pi}{3}n\right)$
 - iii) $x(t) = e^{j\frac{12\pi}{7}t} + e^{j\frac{12\pi}{5}t}$ (10 Marks)
- b. Calculate the energy and power of the following signals as applicable:
 - i) $x[n] = (j)^n + (j)^{-n}$
 - ii) $x[n] = 8(0.5)^n U[n]$ (06 Marks)
- c. Determine and sketch the even and odd parts of the signal depicted in Fig.Q.1(c). (04 Marks)

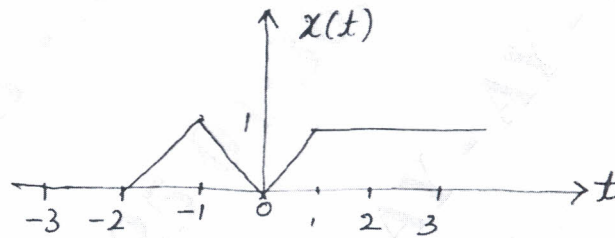


Fig.Q.1(c)

OR

- 2 a. A trapezoidal pulse $x(t)$ is defined by

$$x(t) = \begin{cases} 5-t, & 4 \leq t \leq 5 \\ 1, & -4 \leq t \leq 4 \\ t+5, & -5 \leq t \leq -4 \\ 0, & \text{otherwise} \end{cases}$$

is applied to a differentiator having the input-output relation $y(t) = \frac{dx(t)}{dt}$. Find the energy of $y(t)$. (04 Marks)

- b. Show that the product of two even signals or two odd signals is an even signal, while the product of an even and odd signal is an odd signal. (06 Marks)

c. From the signals indicated in Fig.Q.2(c), derive the following signals:

- i) $x(t-1) y(-t)$
- ii) $x(t) [\delta(t-1) + \delta(t-2)] + y(t) s(t+2)$
- iii) $x(t) y(t-1)$
- iv) $x(t) y(-t-1)$

(10 Marks)

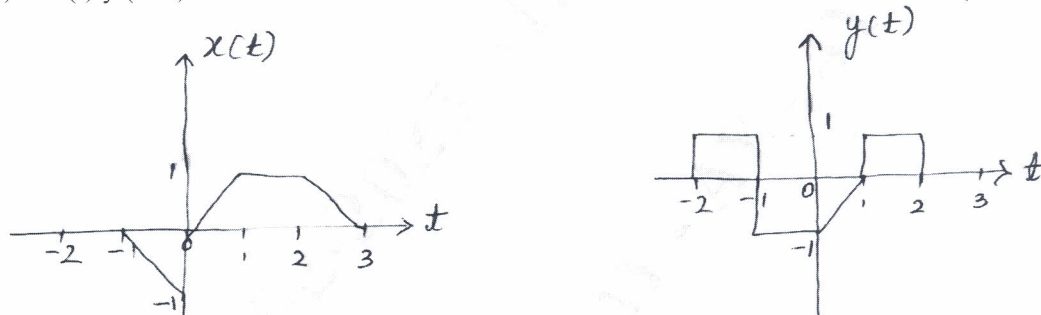


Fig.Q.2(c)

Module-2

- 3 a. Determine whether the system described by the following input-output relation is
 i) Linear ii) Causal iii) Time-invariant iv) Memoryless v) Stable.

$$y(t) = 2x(t) + 3$$

(06 Marks)

- b. Perform the convolution operation on the following signals:

$$x(t) = e^{-2t} u(t)$$

$$h(t) = \mu(t+2)$$

(08 Marks)

- c. Show that

$$i) [x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

$$ii) x(n) * h(n) = h(n) * x(n)$$

(06 Marks)

OR

- 4 a. Given $x[n] = \alpha^n U[n]$ and $h[n] = \beta^n U[n]$, perform $x[n] * h[n]$. (08 Marks)

- b. Determine whether the following systems represented by input-output relations are invertible. If invertible then represent their inverse system

$$i) y(t) = s^{to} \{x(t)\} \quad ii) y(t) = \frac{1}{L} \int_{-\infty}^t x(z) dz$$

$$iii) y(t) = x^2(t) \quad iv) y(t) = 2x(t)$$

(08 Marks)

- c. Perform convolution operations on the following signals and sketch the output:

$$x[n] = \delta(n+1) + 2\delta(n) + 3\delta(n-1) - 2\delta(n-2) + \delta(n-3)$$

$$h[n] = \left(\frac{1}{2}\right)^n [U[n] - U[n-4]]$$

(04 Marks)

Module-3

- 5 a. Determine whether the following systems represented by impulse responses are stable, causal and memoryless.

$$i) h(n) = U(n-1) - U(n-5)$$

$$ii) h(t) = e^{-t} \mu(+t)$$

$$iii) h(n) = 0.5^{|n|}$$

$$iv) h(t) = \mu(t-1)$$

(08 Marks)

- b. Show that step response of an LTI system is running integral of impulse response. (04 Marks)
- c. Evaluate the Fourier series representation for the signal $x(t) = \sin(2\pi t) + \cos(3\pi t)$. Sketch the magnitude and phase spectra. (08 Marks)

OR

- 6 a. State and prove convolution property for continuous time Fourier series. (04 Marks)
- b. Find the Fourier series representation for the given signal and draw its magnitude and phase spectra. (10 Marks)

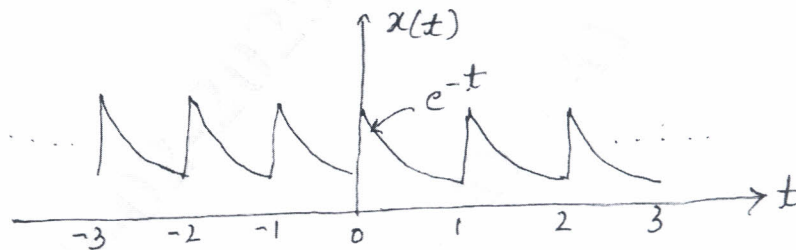


Fig.Q.6(b)

- c. Find the unit step response of the systems given by their impulse responses

i) $h(n) = \left(\frac{1}{2}\right)^n U(n)$ ii) $h(t) = e^{-|t|}$ (06 Marks)

Module-4

- 7 a. State and prove the following properties applicable to continuous time Fourier transform
i) Time shift ii) Frequency differentiation (06 Marks)
- b. Find the continuous time Fourier transform of $x(t) = e^{-at} u(t)$; $a > 0$. Draw its magnitude and phase spectra. (08 Marks)
- c. Find the discrete time Fourier transform of the following signals:
i) $x(n) = (-1)^n U(n)$
ii) $x(n) = a^{|n|}$. (06 Marks)

OR

- 8 a. State and prove Parseval's theorem with respect to discrete time Fourier transform and indicate the importance of it. (06 Marks)
- b. The discrete time Fourier transform of a real signal $x(n)$ is $X(\Omega)$. How is the discrete time Fourier transform of the following signals related to $X(\Omega)$.
i) $y_1(n) = x(-n)$
ii) $y_2(n) = x(n) * x(-n)$
iii) $y_3(n) = (1 + \cos n\pi) x(n)$
iv) $y_4(n) = (-1)^{n/2} x(n)$. (08 Marks)
- c. Find the continuous time Fourier transform of
i) $x(t) = \cos(\omega_0 t)$
ii) $x(t) = \begin{cases} 1; & -T < t < T \\ 0; & \text{otherwise} \end{cases}$ (06 Marks)

Module-5

- 9 a. Find the Z-transform of the following sequences and plot its ROC.

i) $x(n) = \left(\frac{1}{2}\right)^n U(n-2)$

ii) $x(n) = 3\left(\frac{-1}{2}\right)^n U(n) - 2[3^n U(-n-1)]$ (06 Marks)

- b. If $x(n)$ is causal, then prove that

i) $x(0) = \lim_{Z \rightarrow \infty} Z X(z)$

ii) $x(\infty) = \lim_{Z \rightarrow 1} [X(Z)(Z-1)]$ (08 Marks)

- c. A LTI system is given by the system function

$$H(Z) = \frac{3 - 4Z^{-1}}{(1 - 3.5Z^{-1} + 1.5Z^{-2})}$$

Specify the ROC of $H(z)$ and $h(n)$ for the following conditions:

- i) The system is stable
ii) The system is causal.

(06 Marks)

OR

- 10 a. Use the properties of Z-transform to find the Z-transform of the following:

i) $a^{-n} U(-n)$

ii) $\left(\frac{1}{2}\right)^n U(n) * \left(\frac{1}{3}\right)^n U(n)$

iii) $\left(\frac{1}{3}\right)^n U(n) + \left(-\frac{1}{2}\right)^n U(n)$ (08 Marks)

- b. Find the inverse Z-transform of

$$X(Z) = \frac{Z(Z^2 - 4Z + 5)}{(Z-3)(Z-2)(Z-1)}$$
 for the ROC $2 < |Z| < 3$ using partial fraction method. (06 Marks)

- c. Determine the impulse response $h(n)$ and system function $H(z)$ of the system that gives the output

$$y(n) = \left(\frac{1}{3}\right)^n U(n) \text{ for an input } x(n) = \left(\frac{1}{2}\right)^n U(n) - \frac{1}{4} \left(\frac{1}{2}\right)^{n-1} U(n-1). \quad (06 \text{ Marks})$$

* * * * *