

Fourth Semester B.E. Degree Examination, Dec.2024/Jan.2025
Engineering Statistics and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Define CDF of a Random variable. Mention its properties and types. (10 Marks)
- b. Given the data in the Table Q1(b)
- Plot the PDF and CDF of the discrete random variable
 - Write expressions for $f_X(x)$ and $F_X(x)$ using unit delta functions and unit step functions :

X	x_a	x_b	x_c	Total
$P[X = x]$	0.24	0.32	0.44	1

Table Q1(b)

(10 Marks)

OR

- 2 a. Summarize the properties of PDF. Prove that the total area under PDF curve is unity. (10 Marks)
- b. Given the data in the Table Q2(b).
- What are the mean and variance of 'X'
 - If $Y = X^2 + 2$, what are μ_y and σ_y^2 .

K	1	2	3	4	5
X_k	2.1	3.2	4.8	5.4	6.9
$P(x_k)$	0.21	0.18	0.2	0.22	0.19

Table Q2(b)

(10 Marks)

Module-2

- 3 a. Explain the following with respect to Bivariate Random variable.
- Correlation
 - Covariance
 - Uncorrelated X and Y
 - Orthogonal X and Y
 - Independent X and Y.
- (10 Marks)
- b. Let X is a random variable, $\mu_x = 4$ and $\sigma_x = 5$ and Y is a random variable, $\mu_y = 6$ and $\sigma_y = 7$. The correlation coefficient is -0.7. If $U = 3x + 2y$, what are i) Var [U] ii) CoV [UX] iii) CoV [UY]. (10 Marks)

OR

- 4 a. Briefly explain the following random variables
- Chi-square RV
 - Student-T RV
 - Cauchy RV
 - Rayleigh RV.
- (10 Marks)
- b. The joint PDF $f_{XY}(x, y) = C$, a constant when $(0 < x < 3)$ and $(0 < y < 3)$ and is '0' otherwise.
- What is the value of constant C
 - What is the PDF's for X and Y
 - What is $F_{XY}(x, y)$ when $(0 < x < 3)$ and $(0 < y < 3)$
 - What are $F_{XY}(x, \infty)$ and $F_{XY}(\infty, y)$
 - Are X and Y independent?
- (10 Marks)

Module-3

- 5 a. Interpret the following with respect to random process i) Random process ii) Ensemble iii) PDF iv) Independence v) Expectations vi) Stationary. (12 Marks)

- b. The magnitude of a zero mean white noise spectrum is $K = 3.6 \times 10^{-8} \text{ V}^2\text{-S}$. This noise is the input to a low pass RC circuit: $R = 38 \text{ k}\Omega$, $C = 0.1 \text{ }\mu\text{F}$. Find the networks output PSD, $S_y(w)$. (08 Marks)

OR

- 6 a. Discuss the Auto correlation and cross correlation functions with their properties. (12 Marks)
- b. A Random process is described by $y(t) = A \cos (w_c t + \theta)$ where A and w_c are constants, but θ is a random variable distributed uniformly between $\pm\pi$. Determine :
- PDF of random variable ' θ '
 - Mean of $y(t)$
 - Auto correlation function $R_y(\tau)$
 - Mean power and Auto variance of $y(t)$

(08 Marks)

Module-4

- 7 a. Illustrate vector space with its properties in detail. (08 Marks)

- b. Apply Gram-Schmidt process to

$$a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Write the result in the form of $A = QR$.

(12 Marks)

OR

- 8 a. Outline the four fundamental subspaces of matrices. (08 Marks)

- b. Determine : i) matrix U and Rank ii) $\text{rref}(R)$ iii) Null space of matrix and identify free

variables in null space for the matrix given $A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$ (12 Marks)

Module-5

- 9 a. Define determinants with its properties in detail. (13 Marks)

- b. Determine the Eigen values of matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (07 Marks)

OR

- 10 a. Diagonalize the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and hence find A^4 , Also find the matrix ' P ' such that $P^{-1}AP$ is diagonal. (14 Marks)

- b. Reduce the matrix A to U and find $\det A$ using pivots of A .

$$A = \begin{bmatrix} 2 & 5 & 3 \\ 1 & 2 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

(06 Marks)
