VISVESVARAYATECHNOLOGICALUNIVERSITY

BELAGAVI

MATHEMATICS HANDBOOK

III and IV Semester BE Program

(Common to all Lateral Entry MATDIP Students)

(FOR SYLLABUS SCHEMES 2002/2006/2010/2015/2017/2018)

Complex Number

A number of the form $a + ib$, where a and b are real numbers, is called a complex number, a is called the real part and b is called the imaginary part of the complex number.

Equality of complex numbers$a + ib = c + id \Leftrightarrow a = c \text{ and } b = d$

Addition of complex numbers $(a + ib) + (c + id) = (a + c) + i\,(b + d)$

Subtraction of complex numbers $(a + ib) - (c + id) = (a - c) + i\,(b - d)$

Multiplication of complex numbers $(a + ib).(c + id) = (ac - bd) + i\,(ad + bc)$

Division of complex numbers $\frac{a+ib}{c+id} = \frac{a+ib}{c+id}.\frac{c-id}{c-id} = \frac{ac+bd}{c^2+d^2} + i\,\frac{bc-ad}{c^2+d^2}$

Polar form of complex numbers $a + ib = r(\cos\theta + i\sin\theta)$

For any non-zero complex number $z = a + ib\,(a \neq 0, b \neq 0)$, there exists thecomplex number $\frac{a}{a^2+b^2} + i\,\frac{-b}{a^2+b^2}$, denoted by$\frac{1}{z}$ or z^{-1}called multiplicative inverse of z such that$(a + ib)\,\frac{a}{a^2+b^2} + i\,\frac{-b}{a^2+b^2} = 1 + i0 = 1$

For any integer k,$i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$.

The conjugate of the complex number $z = a + ib$, denoted by $\bar{z}$, is given by

$\bar{z} = a - ib.$

The plane having a complex number assigned to each of its point is called the complex plane or the Argand plane.

The Argand plane, the $|x + iy| = \sqrt{x^2 + y^2}$.

n^th Derivatives of Standard Functions:

$$D^n[(ax+b)^m] = m(m-1)(m-2)\ldots(m-n+1)(ax+b)^{m-n}.a^n.$$

$$D^n[(ax+b)^n] = n!\,a^n$$

$$D^n[x^n] = n!$$

$$D^n\left[\frac{1}{ax+b}\right] = \frac{(-1)^n n!\,a^n}{(ax+b)^{n+1}}$$

$$D^n[\log(ax+b)] = \frac{(-1)^{n-1}(n-1)!\,a^n}{(ax+b)^n}$$

$$D^n[a^{mx}] = a^{mx}(mloga)^n$$

$$D^n[e^{ax}] = a^n e^{ax}$$

$$D^n[\sin(ax+b)] = a^n sin(ax+b+\frac{n\pi}{2})$$

$$D^n[\cos(ax+b)] = a^n cos(ax+b+\frac{n\pi}{2})$$

$$D^n[e^{ax}\sin(ax+b)] = (a^2+b^2)^{n/2}e^{ax}sin\left(bx+c+ntan^{-1}\left(\frac{b}{a}\right)\right)$$

$$D^n[e^{ax}\cos(ax+b)] = (a^2+b^2)^{n/2}e^{ax}cos\left(bx+c+ntan^{-1}\left(\frac{b}{a}\right)\right)$$

Polar Coordinates and Polar Curves:

Angle between Polar vector and Tangent

$tan\emptyset = r\frac{d\theta}{dr}$ or $cot\emptyset = \frac{1}{r}\frac{dr}{d\theta}$

Angle of Intersection of the Curves

$$|\emptyset_1 - \emptyset_2| = tan^{-1}\left\{\left|\frac{tan\emptyset_1 - tan\emptyset_2}{1 + tan\emptyset_1 . tan\emptyset_2}\right|\right\}$$

Orthogonal Condition

$|\emptyset_1 - \emptyset_2| = \frac{\pi}{2}$ or $tan\emptyset_1 . tan\emptyset_2 = -1$

Series Expansion:

Taylor's Series expansion about the point $x = a$

$y(x) = y(a) + \frac{(x-a)^1}{1!}y'(a) + \frac{(x-a)^2}{2!}y''(a) + \frac{(x-a)^3}{3!}y'''(a) + \cdots.$

Maclaurian's Series expansion about the point $x = 0$

$$y(x) = y(0) + \frac{x^1}{1!}y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y''''(0) + \cdots.$$

Euler's theorem on Homogeneous Function and Corollary

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xyy^2\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

Composite Function

If $z = f(x, y)$ and $x = \emptyset(t)$, $y = \psi(t)$ then $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

If $z = f(x, y)$ and $x = \emptyset(u, v)$, $y = \psi(u, v)$then

$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$ and $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$

If $u = f(r, s, t)$ and $r = g(x, y, z)$, $s = h(x, y, z)$, $t = i(x, y, z)$ then

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x},$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial y}, \qquad \frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial z} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial z}.$$

Jacobians $J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$ and $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$

Reduction Formulae

$$\int_0^{\frac{\pi}{2}} cos^n x dx = \int_0^{\frac{\pi}{2}} sin^n x dx = \begin{cases} \frac{(n-1)(n-3)(n-5)\dots 1}{n(n-2)(n-4)\dots 2}\frac{\pi}{2} \text{ when n is even} \\ \frac{(n-1)(n-3)(n-5)\dots 3}{n(n-2)(n-4)\dots 1}\ 1 \text{ when n is odd} \end{cases}$$

$$\int_0^{\frac{\pi}{2}} sin^m x cos^n x dx = \begin{cases} \frac{(m-1)(m-3)\dots(n-1)(n-3)\dots}{(m+n)(m+n-2)(m+n-4)\dots 2}\frac{\pi}{2} \text{when m } \& \, n \, is \, even \\ \frac{(m-1)(m-3)\dots(n-1)(n-3)\dots}{(m+n)(m+n-2)(m+n-4)\dots} \text{otherwise} \end{cases}$$

Multiple Integrals

Area $A = \iint_A dxdy$- Cartesian form

Area $A = \iint_A rdrd\theta$ - Polar form

Volume $V = \iiint_A dxdydz$- Cartesian form

Volume$V = \iint_A zdxdy$ – by double integral

Gamma Function: $\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx = 2\int_0^\infty e^{-t^2}t^{2n-1}dt$

Beta Function: $\beta(m,n) = \int_0^1 x^{n-1}(1-x)^{m-1}dx = 2\int_0^{\frac{\pi}{2}} sin^{2n-1}\theta\ cos^{2m-1}\theta\ d\theta$

Beta and Gamma relation $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ and $\Gamma(1/2) = \sqrt{\pi}$.

Vector Calculus

Position Vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Magnitude $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

Dot Product of unit vectors $\hat{\imath}.\hat{\imath} + \hat{\jmath}.\hat{\jmath} + \hat{k}.\hat{k} = 1$ and $\hat{\imath}.\hat{\jmath} + \hat{\jmath}.\hat{k} + \hat{k}.\hat{\imath} = 0$

Cross Product of unit vectors $\hat{\imath}\times\hat{\imath} = \hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = 0$ and $\hat{\imath}\times\hat{\jmath} = \hat{k}$, $\hat{\jmath}\times\hat{k} = \hat{\imath}$, $\hat{k}\times\hat{\imath} = \hat{\jmath}$.

Angle between two vectors $cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$; $sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$

Unit vector $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

Velocity $\vec{V} = \frac{ds}{dt}$

Acceleration $\vec{a} = \frac{d^2s}{dt^2}$

For any vectors $\vec{A} = (a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k})$, $\vec{B} = (a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k})$ &

$\vec{C} = (a_3\hat{\imath} + b_3\hat{\jmath} + c_3\hat{k})$

Dot product of two vectors $\vec{A}.\vec{B} = a_1a_2 + b_1b_2 + c_1c_2$

Cross product of two vectors $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Scalar triple product $\vec{A}.(\vec{B} \times \vec{C}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Vector Calculus

Velocity $\vec{v}(t) = \frac{d\vec{r}}{dt}$

Acceleration $\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

The unit tangent vector $\hat{T} = \frac{d\vec{r}}{dt} \Big/ \left|\frac{d\vec{r}}{dt}\right|$

Angle between the tangents $cos\theta = \frac{\vec{T_1}.\vec{T_2}}{|\vec{T_1}||\vec{T_2}|}$

Component of velocity $C.V = \vec{v}.\hat{n}$, where $\hat{n}$ is the unit vector

Component of accelerations $C.A = \vec{a}.\hat{n}$

Tangent component of acceleration $T.C.A = \vec{a}.\vec{v} / |\vec{v}|$

Normal Component of acceleration:

$$N.C.A = \left|\vec{a} - (tangential\ component) \times \left(\vec{v}/_{|\vec{v}|}\right)\right|$$

Gradient of $\emptyset$: $grad\emptyset = \nabla\emptyset = \frac{\partial\emptyset}{\partial x}\hat{\imath} + \frac{\partial\emptyset}{\partial y}\hat{\jmath} + \frac{\partial\emptyset}{\partial z}\hat{k}$

Unit vector normal to the surface: $\hat{n} = \frac{\nabla\emptyset}{|\nabla\emptyset|}$

Directional derivative along vector $\vec{n}$ $is\ \ D.D = \nabla\emptyset.\hat{n}$

Angle between the surfaces $cos\theta = \frac{\nabla\emptyset_1.\nabla\emptyset_2}{|\nabla\emptyset_1||\nabla\emptyset_2|}$

Divergence of vector field $\vec{F} = f_1 i + f_2 j + f_3 k$

$$div\,\vec{F} = \nabla.\vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

Curl of vector field $\vec{F}$: $curl\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$

Solenoidal vector field: $div\vec{F} = \nabla.\vec{F} = 0$

Irrotational vector field: $curl\vec{F} = \nabla \times \vec{F} = 0$

List of vector identities

$curl(grad\emptyset) = \nabla \times \nabla\emptyset = 0$

$div(curl\vec{F}) = \nabla.(\nabla \times \vec{F}) = 0$

$div(\emptyset\vec{F}) = \emptyset(div\vec{F}) + grad\emptyset.\vec{F}$

$curl(\emptyset\vec{F}) = \emptyset(curl\vec{F}) + grad\emptyset \times \vec{F}$

Linear Algebra

Inverse of square matrix A: $A^{-1} = \frac{(adjA)}{|A|}$

Rank of a matrix A: The number of non-zero rows in the echelon form of A is equal to rank of A.

Normal form of a matrix: $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

Gauss Elimination method

The system is reduced to upper triangular system from which the unknowns are found by back substitutions.

Eigenvalues

Roots of the characteristic equation $|A - \lambda I| = 0$.

Eigen Vectors

Non-zero solution $x = x_i$ of $|A - \lambda I|x = 0$.

Diagonal form

$$D = P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Nature, Rank and Index of Quadratic forms:

- **Positive-definite** if all the Eigen values of A are positive.
- **Positive-semi definite** if all the Eigen values of A are non-negative and at least one of the eigenvalues is zero.
- **Negative-definite** if all the Eigen values of A are negative.
- **Negative-semi definite** if all the Eigen values of A are negative and at least one of the Eigen values is zero.
- **Indefinite** if the matrix A has both positive and negative eigenvalues.
- **Rank** the number of non-zero terms.
- **Index** the number of positive terms.
- **Signature** the number of positive terms minus the number of negative terms.