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18CS36

Third Semester B.E. Degree Examination, June/July 2025
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Establish the following logical equivalence using laws of logic
$(p \vee q) \vee (\sim p \wedge \sim q \wedge r) \Leftrightarrow p \vee q \vee r$. **(06 Marks)**

b. Define converse, inverse and contra-positive of a conditional statement. Hence write down the converse, inverse and contra-positive of "If you don't wear a mask, then you will be punished". **(07 Marks)**

c. Write the following in symbolic from and establish if the argument is valid: "If A gets the supervisors position and work hard, then he will get a raise. If he gets a raise then he will buy a new car. He didn't buy a new car. Therefore A did not get supervisor's position or he did not work hard". **(07 Marks)**

OR

2 a. Prove that for any propositions p, q, r the compound propositions,
$[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a Tautology. **(06 Marks)**

b. Verify whether the following argument is valid :
$\forall x, p(x) \vee q(x)$
$\exists x, \sim p(x)$
$\forall x, \sim q(x) \vee r(x)$
$\forall x, s(x) \to \sim r(x)$
$\therefore \exists x, \sim s(x)$ **(07 Marks)**

c. i) Prove by direct method :
"if n is an even integer then n^2 is also even"
ii) Prove by indirect method :
"for all real numbers x and y if $x + y > 100$ then $x > 50$ or $y > 50$" **(07 Marks)**

Module-2

3 a. Prove by mathematical Induction :
$\sum_{i=1}^{n} i\, 2^{i} = 2 + (n-1)2^{n+1} \quad \forall n \in z^{+}$. **(06 Marks)**

b. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 50,00,000? **(07 Marks)**

c. Find the coefficient of $x^{11}y^4z^2$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$. **(07 Marks)**

OR

4 a. How many arrangements are there for the letters of the word "SOCIOLOGICAL"? In how many of the arrangements. **(06 Marks)**
i) A and G are adjacent ii) All the vowels are adjacent

b. A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in Part C. It is required to answer seven questions selecting atleast two questions from each part. In how many ways can a student select his 7 questions for answering? **(07 Marks)**

c. In how many ways one can distribute 8 identical items in 4 distinct containers so that the fourth container has an odd number of items in it. **(07 Marks)**

Module-3

5 a. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aRb of and only if a is a multiple of b
i) Write R as a set of ordered pairs
ii) Represent R as a matrix
iii) Draw the digraph of R **(06 Marks)**

b. For a fixed integer n >1, prove that the relation "congruent modulo n" is an equivalence relation on the set of all integer z. **(07 Marks)**

c. Prove the following :
A function $f : A \rightarrow B$ is invertible if and only if it is one-to-one and onto. **(07 Marks)**

OR

6 a. State Pigeon hole principle. Prove that if 30 dictionaries contain a total of 61,327 pages then atleast one of the dictionary must have atleast 2045 pages. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 5, 6, 7} and R be the equivalence relation on A that induces the partition A = {1, 2} ∪ {3} ∪ {4, 5, 7} ∪ {6}, find R. **(07 Marks)**

c. Let A {1, 2, 3, 6, 9, 18} and define R on A by "xRy iff x divides y". Draw the Hasse diagram of the poset. Also write the matrix of relation. **(07 Marks)**

Module-4

7 a. In how many ways the 26 letters of English alphabet are permuted so that none of the patterns SPIN, GAME, PATH or NET occurs. **(06 Marks)**

b. Define derangements. Find the number of derangements of the numbers 1, 2, 3 and 4. List all the derangements. **(07 Marks)**

c. An apple, a banana, a mango and an orange are to be distributed among four boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish to have apple. The boy B_3 does not want banana or mango and B_4 returns the orange. In how many ways the distribution made so that no boy is displeased? **(07 Marks)**

OR

8 a. How many integers between 1 and 300 (inclusive) are divisible by
i) atleast one of 5, 6, 8 ii) None of 5, 6, 8? **(06 Marks)**

b. Find the root polynomial of the following board

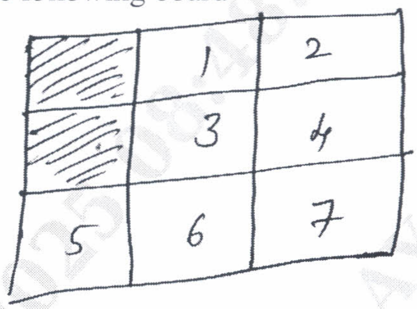


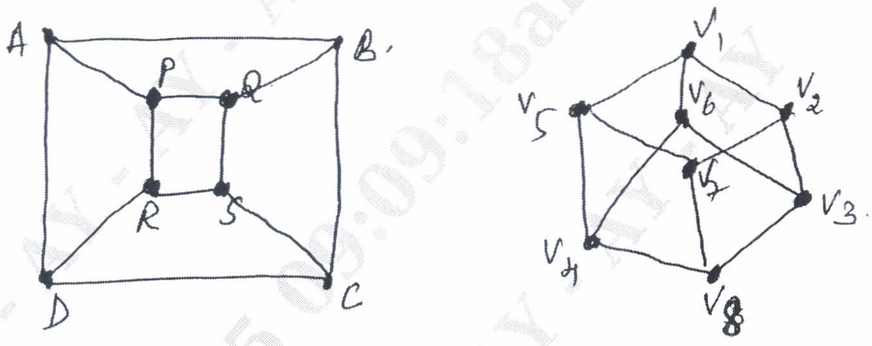
Fig Q8(b) **(07 Marks)**

c. If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 37$. Satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $n \geq 0$, determine the constants b and c and then solve the relation for a_n. **(07 Marks)**

Module-5

9 a. Define the terms : i) regular graph ii) trail iii) complete graph iv) connected graph, with one example for each. **(06 Marks)**

b. Define isomorphism. Verify that the following two graphs are isomorphic.



(07 Marks)

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(07 Marks)**

OR

10 a. Define the terms :
i) Bipartite graph ii) Subgraph iii) Complement of a graph iv) Complete binary tree, with one example for each. **(06 Marks)**

b. Define a tree. Prove that a tree T with n vertices has n-1 edges. **(07 Marks)**

c. Obtain optimal prefix code for the message "ROAD IS GOOD" using labelled binary tree. Indicate the code. **(07 Marks)**

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