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Nonlinear Observers for Data Fusion based on Robustness norm for System with Delay and Missing Measurements

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Abstract: In this paper, H_∞ (H-Infinity, HI)-based nonlinear observers for continuous time dynamic system with state delay, and randomly missing measurements are presented. The Lyapunov energy (LE) functional to derive sufficient conditions for the local asymptotic stability for the observer-state error equations is derived. This observer's performance with and without randomly missing measurements is evaluated by simulations carried out in MATLAB. The results validate the theoretical asymptotic behaviour of the proposed HI-based nonlinear observer. Then, the nonlinear observer is extended to nonlinear system with state delay and randomly missing measurements in the measurement data level (MLF) and state vector fusion (SVF) modes for multi-sensor data fusion (MSDF). It is ascertained that the derived theoretical result automatically extends to these nonlinear observers for data fusion due to their non-complicated structures.

Keywords: Delayed states, randomly missing data, HI-based nonlinear observers, asymptotic result, measurement and state vector level fusion.

1. Introduction

In modelling, control theory & practice, and the estimation-cum-filtering theory, the objective of an observer is to reconstruct the state of a dynamic system using the knowledge of the system input/output (I/O) data. In case of linear systems, the system states can be estimated using the Luenberger observer (LO) or Kalman filter (KF); whereas for a nonlinear system one uses an extended Luenberger observer (ELO) or extended Kalman filter (EKF) [1]. Yet, the results for state estimation of nonlinear systems with delayed states and randomly missing measurements are only very few. However, there are several real-life dynamic systems like transportation, chemical reactors, biological systems, computer networks, communication systems, and wireless sensor networks (WSNs), wherein state delays and/or missing measurements could occur. Sometimes, in a data-communication channel a few or many measurement data might be missing (from one or more sensors); in such cases, it becomes important to evaluate the performance of the data processing algorithms despite of these missing data. In the absence of certain signals during certain time intervals, invariably one would have only the random (measurement) noise or some unknown deterministic disturbance present in these channels. This joint-combined synergy aspect has not evolved much in the context of nonlinear systems. However, some work has been done in certain areas and specific topics [2-10]. In [2-4] a few special cases have been considered: i) intermittent observations, ii) missing data in online condition monitoring of systems, and iii) occasional packet dropouts, etc. In [5], a system with multiple sensor-delay has been considered, but the algorithm happens to be a bit complex. The problem of measurement data outliers and missing data has been considered in [6], but the illustration has been given only for simple time-series case. In addition, although, [7, 8, 10] deal with missing measurements, the aspect of state delay is not treated; whereas in [9] only the system delay is considered. Interestingly in [11] an EKF-based nonlinear observer has been proposed for the system with time-delays and some asymptotic results have been presented [11, 12], but the aspect of missing measurements has not been studied. Here, we consider the state delay as well as randomly missing measurements in a nonlinear observer in a synergistic manner. In particular, a few data might be missing due to: i) a failure of a senor (or more sensors), and/or b) there might be a problem in a communication channel such that the received data are only the noise and the real signal is missing. Hence, it is very important to incorporate the situation of missing data in a nonlinear observer in some optimal or sub-optimal way. The (state) time delay also, could be encountered in a few real-time systems, due to latency time of certain channels; and this time delay is a key factor that would influence the overall system's stability and performance. That is, if these system's state delay and missing data are not handled appropriately in a tracking algorithm, then one might lose the track, or the tracking performance might be poor. In certain cases the system's state delay would directly affect the measured data, since these data are dependent on the delayed states in some specific way.

Hence, an HI-based nonlinear observer is proposed that would handle state delay as well as randomly missing data leading to sub-optimal estimator-cum-observer; and also the asymptotic condition for the observer error dynamics based on LE Functional is derived. Then, the performance of the proposed observer is illustrated by implementing the algorithm in MATAB. Also, proposed are the nonlinear observer structures for nonlinear continuous time system with state delay and randomly missing measurements in the context of MSDF; for MLF, and SVF fusion, and ascertain that a state vector fusion formula for this observer can be directly taken from that for the KF/EKF, and that the asymptotic result also extends to these fusion mode-observers in a straight forward manner. Thus, the presented results and inferences here are novel in the area of observer and multisensory data fusion theory.

2. Nonlinear system and observer error dynamics

Let the nonlinear delayed state model with randomly missing measurements be given as

$$\begin{aligned} \dot{x}(t) &= f(x(t), x(t-\tau), u(t)) + g(x)w(t) \\ z &= h_1(x); \quad y(t) = \beta h_2(x) + q(x)w(t) \end{aligned} \qquad (1)$$

In (1), the scalar quantity β is a Bernoulli sequence (in which case one could have written y(k)= β(k)H(k)x(k), after linearization of h_2), and this sequence would take values *0* and *1* randomly; thus, $E\{\beta(k)=1\} = b(k)$, and $E\{\beta(k)=0\} = 1-b(k)$, with b as the percentage of measurement data that arrive to/from a sensor node, and E{.} is the mathematical expectation. This arrangement signifies that a few measurements data are randomly missing, and constant *b* is assumed to be known/pre-specified. The initial conditions for the states and delayed state are assumed to be appropriately specified [13]. The variables in (1), have usual appropriate dimensions, and presently consider that these belong to real 2D space (say H_2 vector spaces), however, in the context of HI theory these variables can be considered as generalized random variables. In (1), w(.) is considered as an unknown disturbance, y(.) is a measurement vector, and z(.) is the quantity to be estimated [13], and often it is called estimatee of the state vector x. It is also, assumed that the nonlinear function f, g, h_1, h_2, and q are continuously differentiable. Presume that the system of (1) has a unique solution. Then, a nonlinear observer for the system of (1) is proposed as

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}(t), \hat{x}(t-\tau), u(t)) + L(t)(y(t) - \hat{y}(t)) \\ \hat{z}(t) &= h_1(\hat{x}); \quad \hat{y}(t) = \beta h_2(\hat{x}) \end{aligned} \qquad (2)$$

In (2), L(t) is an observer gain matrix of appropriate dimension, and is determined by using the cost function based on H-Infinity concept. The idea is to seek that the L_2 gain from the input disturbance energy to the output estimation error energy obeys the following inequality [13]

$$\int_{t_0}^{T} \|\tilde{z}(\tau)\|^2 \, d\tau \le \int_{t_0}^{T} \|w(\tau)\|^2 \, d\tau; \; T > t_0 \qquad (3)$$

In (3), the estimation error defined as

$$\tilde{z}(t) = z(t) - \hat{z}(t) = h_1(x) - h_1(\hat{x}) \qquad (4)$$

The cost function for (3) can be formulated as

$$J(w, L) \square \frac{1}{2}\int_{t_0}^{T} (\|\tilde{z}(t)\|^2 - \gamma^2 \|w(t)\|^2) dt \qquad (5)$$

In (5), γ (>0) specifies the error energy gain from the input to the output and can be considered as a tuning parameter. This means that the actual error energy gain is bounded from above by square of this factor, so the observer is not optimal but it is sub-optimal, and yet robust. Finally following [13], the observer gain that includes the missing data factor can be chosen as

$$L(t) = bP(t)H^T(t) \qquad (6)$$

In (5), P(t) is obtained as the solution of the observer Riccati differential (ORD) equation

$$\begin{aligned} \dot{P}(t) = {} & P(t)A_0(t) + A_0^T(t)P(t) + P(t)[\frac{1}{\gamma^2}C^T(t)C(t) - L^T(t)L(t)] \\ & \cdot P(t) + A_1(t)A_1^T(t) + B(t)B^T(t); \\ & with \; P(0) = P_0; P(T) = 0 \end{aligned} \qquad (7)$$

Also, in (6) matrix R that is found in KF gain formula ($P(t)H^T(t)R^{-1}$) is not present; however it can be easily incorporated for the sake of tuning the observer, and can be regarded as a weighting matrix/factor. In that case the observer gain would be $L(t) = bP(t)H^T(t)R^{-1}$, and this would not pose any problem in the proposed derivation of the asymptotical result for this nonlinear observer. The alternative form of the ORD equation (7) is [13]

$$\begin{aligned} \dot{P}(t) = {} & P(t)A_0^T(t) + A_0(t)P(t) + P(t)[\frac{1}{\gamma^2}C^T(t)C(t) - b^2H^T(t)H(t)] \\ & \cdot P(t) + A_1(t)A_1^T(t) + B(t)B^T(t) \end{aligned} \qquad (8)$$

Now, the observer gain (6) is based on (7) and hence, in turn on the H infinity norm-cum-filtering theory [13]. Various Jacobians needed in (8) are obtained as

$$A_0(t) = \frac{\partial f(.)}{\partial \hat{x}(t)}; \quad A_1(t) = \frac{\partial f(.)}{\partial \hat{x}(t-\tau)};$$

$$B(t) = \frac{\partial g(.)}{\partial \hat{x}(t)}; \; C(t) = \frac{\partial h_1(.)}{\partial \hat{x}(t)}; \; H(t) = \frac{\partial h_2(.)}{\hat{x}(t)}; D(t) = \frac{\partial q(.)}{\partial \hat{x}(t)} \qquad (9)$$

In (8), an additional term, $A_1(.)$ arises due to the state delay as in (2). In (6)-(8), one can see that although, various quantities are function of ($\hat{x}$, t), the dependence only on t for simplicity is mentioned. By subtracting (2) from (1) one obtains the following error dynamics

$$\begin{aligned} \dot{e}(t) = {} & A_0(t)e(t) + A_1(t)e(t-\tau) - L(t)H(t)b(x(t) - \hat{x}(t)) \\ & + \phi(.) + (B(t)x - L(t)D(t)x)w \\ = {} & A_0(t)e(t) + A_1(t)e(t-\tau) - bL(t)H(t)e(t) \\ & + \phi(.) + (B(t) - L(t)D(t))xw \end{aligned} \qquad (10)$$

Then drop the last term from (10), to obtain the error dynamics for the observer without the disturbance term. In (10), the nonlinear function is

$$\begin{aligned} \phi(.) = {} & -A_0(t)e(t) - A_1(t)e(t-\tau) \\ & + f(x,t,t-\tau) - f(\hat{x},t,t-\tau) \end{aligned} \qquad (11)$$

The form of (10) is obtained by adding and subtracting the terms related to the Jacobians in the original equations of the error dynamics. In (11) the full forms for f and ϕ (.) are given for clarity

$$\phi(.) = \phi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau)) \qquad (12)$$

$$\begin{aligned} f(x,t,t-\tau) &= f(x(t), x(t-\tau)) \\ f(\hat{x},t,t-\tau) &= f(\hat{x}(t), \hat{x}(t-\tau)) \end{aligned} \qquad (13)$$

However, for simplicity one can use very compact form of f by avoiding the arguments since, these would be implied any way from its defining form (1), and (2), and also u(t) is omitted. The state errors are given as

$$e(t) = x(t) - \hat{x}(t); \; e(t-\tau) = x(t-\tau) - \hat{x}(t-\tau) \qquad (14)$$

3. Asymptotic result for observer error dynamics

Let us consider the following conditions [11,12] for the local asymptotic behaviour of the observer error dynamics of (10)

1. The solution of the ORD equation (8) is considered bounded

$$p_l I \leq P(t) \leq p_u I \tag{15}$$

with $p_l, p_u > 0$ as positive constants (since P(t) is also theoretically, positive definite and symmetric matrix), and p_l, p_u are the lower and upper bounds respectively; and I is the identity matrix.

2. The nonlinearity in (11), (12) of the error dynamics is bounded

$$\|\phi(.)\| \leq \rho_1 \|x(t) - \hat{x}(t)\|^2 + \rho_2 \|x(t-\tau) - \hat{x}(t-\tau)\|^2 \tag{16}$$

with the bounding constants equal to or greater than zero. Then, the nonlinear observer error dynamics (10) are locally asymptotically stable, if the conditions 1 and 2 are satisfied.

First, consider the following normalized LE functional to establish the asymptotic stability of the error dynamics (10)

$$V(t) = e^T(t) Y(t) e(t) \tag{17}$$

In (17), Y(t) is the normalizing/weighting matrix and can be recognized as an information matrix, and is given as $Y(t)=P^{-1}(t)$. In the case of KF, P(t) is considered as the covariance matrix of the state error-vector. However, since observer gain is used form from the H infinity theory, and since variable w(.) is considered as some unknown disturbance, then, following the H infinity filter theory, (and in particular for the presented nonlinear observer since, one is not dealing with the stochastic noise processes), one would call the matrix P(t) as the Gramian matrix. For the inverse of matrix P(t), one can still retain the name 'information matrix', or call it as the information Gramian, In that case the variables x(.), z(.), y(.), and e(t) can be considered as the generalized 'random' variables. The LE functional (17) is positive definite because it is governed by the condition 1, the inequality of (15) as follows

$$\frac{1}{p_u} \|e(t)\|^2 \leq e^T(t) Y(t) e(t) \leq \frac{1}{p_l} \|e(t)\|^2 \tag{18}$$

The point in obtaining the asymptotic result is that the time derivative of the LE functional, (17), under the constraints governed by error dynamics (10), and (6) and (8), should be negative definite for all time t. For simplicity, write the error dynamics as

$$\begin{aligned} \dot{e}(t) &= A_0 e(t) + A_1 e(t-\tau) - bLHe(t) \\ &\quad + \phi(.) + (B - LD)xw \end{aligned} \tag{19}$$

Then the time derivative of (17) is given as

$$\dot{V}(t) = e^T(t)\dot{Y}(t)e(t) + e^T(t)Y(t)\dot{e}(t) + \dot{e}^T(t)Y(t)e(t)$$

$$\begin{aligned} \dot{V}(t) &= e^T(t)\dot{Y}(t)e(t) + e^T(t)\{A_0^T Y(t) + Y(t)A_0 \\ &\quad - bH^T L^T Y(t) - bY(t)LH\}e(t) + e^T(t-\tau)A_1^T Y(t)e(t) \\ &\quad + e^T(t)Y(t)A_1 e(t-\tau) \\ &\quad + 2e^T(t)Y(t)\phi(.) + 2e^T(t)Y(t)(B-LD)xw \end{aligned} \tag{20}$$

Next, substitute for $\dot{Y}(t) = -Y(t)\dot{P}(t)Y(t)$, and (6) and (8) in (20) to obtain

$$\begin{aligned} \dot{V}(t) &= -e^T(t)Y(t)A_1(t)A_1^T(t)Y(t)e(t) + e^T(t-\tau)A_1^T(t)Y(t)e(t) \\ &\quad + e^T(t)Y(t)A_1(t)e(t-\tau) - e^T(t-\tau)e(t-\tau) \\ &\quad + e^T(t-\tau)e(t-\tau) - e^T(t)Y(t)BB^T Y(t)e(t) \\ &\quad + \frac{1}{\gamma^2} e(t)C^T Ce(t) - b^2 e^T(t)H^T He(t) \\ &\quad + 2e^T(t)Y(t)\phi(.) + 2e^T(t)Y(t)(B-LD)xw \end{aligned} \tag{21}$$

In obtaining (21), because of the substitution of (6), and (8), several common terms cancel out; no approximations are made. In (21), add and subtract the term $e^T(t-\tau)e(t-\tau)$, and thus due to the structure of the first four terms of (21), one can combine these in the compact form $-\|A_1^T(t)Y(t)e(t) - e(t-\tau)\|^2$, and by using this in (21) one obtains

$$\dot{V}(t) = -\left\| A_1^T(t)Y(t)e(t) - e(t-\tau) \right\|^2 - e^T(t)Y(t)SY(t)e(t) + \frac{1}{\gamma^2} e^T(t)C^T Ce(t) - b^2 e^T(t)H^T He(t) + e^T(t-\tau)e(t-\tau) + 2e^T(t)Y(t)\phi(.) + 2e^T(t)(Y(t)B - bH^T D)xw \quad (22)$$

In (22), S=BB^T, then, get the following equivalent inequality by dropping the first compact term (this does not affect the inequality) from (22)

$$\dot{V}(t) \le -e^T(t)Y(t)SY(t)e(t) + \frac{1}{\gamma^2} e(t)C^T Ce(t) - b^2 e^T(t)H^T He(t) + e^T(t-\tau)e(t-\tau) + 2e^T(t)Y(t)\phi(.) + 2e^T(t)\{Y(t)B - bH^T D)xw \quad (23)$$

Now, in (23), assume that $\left\|C^T C\right\| \le c^2$; $\left\|H^T H\right\| \le h^2$; (c and h being positive constants), $\left\|e(t)\right\|^2 \le \varepsilon^2$, and since, these are known and pre-specified quantities, or should be finite, one obtains

$$\dot{V}(t) \le -e^T(t)Y(t)SY(t)e(t) + \frac{1}{\gamma^2} c^2\varepsilon^2 - b^2h^2\varepsilon^2 + \left\|e(t-\tau)\right\|^2 + 2e^T(t)Y(t)\phi(.) + 2e^T(t)\{Y(t)B - bH^T D\}xw \quad (24)$$

Then, using the inequality from (16), one obtains

$$\dot{V}(t) \le -e^T(t)Y(t)SY(t)e(t) + 2\{\frac{\rho_1}{p_u}\left\|e(t)\right\|\left\|e(t)\right\|^2 + \frac{\rho_2}{p_u}\left\|e(t)\right\|\left\|e(t-\tau\right\|^2\} + \frac{1}{\gamma^2}c^2\varepsilon^2 - b^2h^2\varepsilon^2 + \left\|e(t-\tau\right\|^2 + 2e^T(t)\{Y(t)B - bH^T D\}xw$$

$$\dot{V}(t) \le -\frac{s_l}{p_u^2}\left\|e(t)\right\|^2 + 2\{\frac{\rho_1}{p_u}\left\|e(t)\right\|\left\|e(t)\right\|^2 + \frac{\rho_2}{p_u}\left\|e(t)\right\|\left\|e(t-\tau\right\|^2\} + \frac{1}{\gamma^2}c^2\varepsilon^2 - b^2h^2\varepsilon^2 + \left\|e(t-\tau\right\|^2 + 2e^T(t)\{Y(t)B - bH^T D\}xw \quad (25)$$

$$\dot{V}(t) \le -\{\frac{s_l}{p_u^2} - \frac{2\rho_1}{p_u}\left\|e(t)\right\|\}\left\|e(t)\right\|^2 - \{b^2h^2 - 1 - \frac{1}{\gamma^2}c^2 - \frac{2\rho_2}{p_u}\left\|e(t)\right\|\}\left\|e(t-\tau\right\|^2 + 2e^T(t)\{Y(t)B - bH^T D\}xw \quad (26)$$

In (25), s_l is the smallest (positive) eigenvalue of the matrix S, that is positive definite. For $\left\|e(t)\right\| \le \varepsilon$, the following condition from (26) is obtained

$$\dot{V}(t) \le -\{\frac{s_l}{p_u^2} - \frac{2\rho_1}{p_u}\varepsilon\}\left\|e(t)\right\|^2 - \{b^2h^2 - 1 - \frac{1}{\gamma^2}c^2 - \frac{2\rho_2}{p_u}\varepsilon\}\left\|e(t-\tau\right\|^2 + 2\varepsilon(\frac{B}{p_u} - bH^T D)xw \quad (27)$$

$$\dot{V}(t) \le -\{\frac{s_l}{2\rho_1 p_u}; \frac{(\gamma^2(b^2h^2-1) - c^2)p_u}{2\gamma^2\rho_2}\}\left\|e(t)\right\|^2 \quad (28)$$

In (27), if it is assured that the last term in the parenthesis is zero, by having $B = bH^T Dp_u$ (or alternatively one can heuristically assume that the estimation error, the state and the unknown disturbance are uncorrelated, and drop the last term) and since, all the constants and bounds appearing in the {.;.} in (28) are positive, and also, regarding the bounding constants in (16), as positive, (in the case that these are really equal to zero, one can assign them slightly positive values, without loss of any generality; and since these constants are arbitrary, and one can always assure that $\gamma^2h^2b^2 > \gamma^2 + c^2$). Thus, it is seen that the time derivative of the Lyapunov energy functional is bounded from above as in (28). Since, the Lyapunov energy functional is positive definite as in (17) and (18), and its time derivative is locally negative definite as in (28), the observer error dynamics of the newly proposed HI based nonlinear observer for system with state delay, and randomly missing measurement data is locally asymptotically stable.

4. Nonlinear observers for data fusion schemes

Multi-sensor data fusion is an evolving technology at software, algorithms and hardware levels and is defined recently as:

'an act, that could be additive, multiplicative, operative, and/or logical by which a) the quantitative information in the sense of Fisher's information matrix is enhanced by fusing/combining data from more than one sensor or source, and/or b) the prediction accuracy is enhanced, compared to the usage of a single sensor-data or source' [14].

4.1 Observer for the measurement data level fusion

Consider the model with state delay, and missing data in the measurement data level fusion. The state-space model is similar to (1), and the measurement model for the two-sensor scheme is made simpler and given as

$$\begin{aligned} \dot{x}(t) &= f(x(t), x(t-\tau), u(t)) + g(x)w(t) \\ z &= h_1(x); \quad y^i(t) = \beta^i h_2^i(x) \end{aligned} \tag{29}$$

In (29), $y^i(t)$, i=1,2 is the output from the first (i=1), and the second (i=2) sensors; and here also, the scalar quantity β^i is a Bernoulli sequence; and in which case one could have written y(k)= β^i (k)H(k)x(k), without loss of any generality, and this sequence takes values *0* and *1* randomly; thus, one has $E\{\beta^i(k)=1\} = b^i(k)$ and $E\{\beta^i(k)=0\} = 1-b^i(k)$, with b^i as the percentage of measurement data that arrive to/from any sensor node for further processing by the nonlinear observer. This signifies that a few data are randomly missing. The constant b^i are assumed to be known and pre-specified, or can be obtained from some related previous data processing exercises. Then, a nonlinear observer for the system of (29) can be specified as

$$\begin{aligned} \dot{\hat{x}}_f(t) &= f(\hat{x}_f(t), \hat{x}_f(t-\tau), u(t)) \\ &\quad + L_c(t)(y_{cm}(t) - \hat{y}_c(t)) \\ \hat{y}_c(t) &= \beta_c H_c \hat{x}_f(t) \end{aligned} \tag{30}$$

In (30) the subscript 'f' denotes the fused state obtained as a result of the MLF of the two measurements from the two sensors, combined at the data level, and (30) is rewritten as

$$\begin{aligned} \dot{\hat{x}}_f(t) &= f(\hat{x}_f(t), \hat{x}_f(t-\tau), u(t)) \\ &\quad + L_c(t)(y_{cm}(t) - bH_c\hat{x}_f(t)) \\ \hat{y}_c(t) &= bH_c \hat{x}_f(t) \end{aligned} \tag{31}$$

In (30), y_{cm} and H_c are appropriate composite vectors/matrices which account for the direct data level fusion (MLF) of the measurement data coming from the two sensors, and b^i can be appropriately accounted for. In (30), $L_c(t)$ is the observer gain matrix of appropriate dimension, and is given as per the HI theory as seen in the previous section

$$L_c(t) = P_f(t)H_c^T \tag{32}$$

In (32), $P_f(t)$ is obtained as the solution of the ORD equation

$$\begin{aligned} \dot{P}_f(t) &= P_f(t)A_0^T(t) + A_0(t)P_f(t) \\ &\quad + P_f(t)[\frac{1}{\gamma^2}C^T(t)C(t) - b^2 H_c^T(t)H_c(t)]P_f(t) \\ &\quad + A_1(t)A_1^T(t) + S \end{aligned} \tag{33}$$

By subtracting (30) from (29) one obtains the following error dynamics

$$\begin{aligned} \dot{e}_f(t) &= A_0(t)e_f(t) + A_1(t)e_f(t-\tau) - L_c(t)H_c(t)b(x_f(t) - \hat{x}_f(t)) \\ &\quad + \phi_f(.) + B(t)x_f w \\ &= A_0(t)e_f(t) + A_1(t)e_f(t-\tau) - bL_c(t)H_c(t)e_f(t) \\ &\quad + \phi_f(.) + B(t)x_f w \end{aligned} \tag{34}$$

In (34), 'f' denotes the fused condition, and it does not have any effect on the dimension of the state vector (and hence state error), it reflects the fact that the observer state has the combined effect of two measurements. In (34), one has the nonlinear function as

$$\begin{aligned} \phi_f(.) &= -A_0(t)e_f(t) - A_1(t)e_f(t-\tau) \\ &\quad + f(x_f, t, t-\tau) - f(\hat{x}_f, t, t-\tau) \end{aligned} \tag{35}$$

In (35), the full forms for the nonlinear functions and $\phi_f(.)$ are given for clarity as

$$\phi_f(.) = \phi(x_f(t), x_f(t-\tau), \hat{x}_f(t), \hat{x}_f(t-\tau))$$
$$f(x_f, t, t-\tau) = f(x_f(t), x_f(t-\tau)) \qquad (36)$$
$$f(\hat{x}_f, t, t-\tau) = f(\hat{x}_f(t), \hat{x}_f(t-\tau))$$

Note that the state errors are given as

$$\begin{aligned} e_f(t) &= x_f(t) - \hat{x}_f(t); \\ e_f(t-\tau) &= x_f(t-\tau) - \hat{x}_f(t-\tau) \end{aligned} \qquad (37)$$

One can observe from (32)-(34), and the bounding conditions of (15), and (16), that the theoretical development of Section 3 is equally applicable to the observer error dynamics of (34), and hence, one infers and ascertains by induction that observer error dynamics of the newly proposed nonlinear observer (31) (based on H infinity filter theory) for systems with state delay, and randomly missing measurement data in the data level fusion is locally asymptotically stable.

4.2 Observer scheme for the state vector fusion

Consider, in the state vector level fusion (SVF), that the measurements coming from two sensors are separately processed at each local sensor node, and then the estimated state vector is obtained from the individual estimates by the SVF formula. Consider the nonlinear dynamics model as in (29)

$$\begin{aligned} \dot{x}^i(t) &= f^i(x^i(t), x^i(t-\tau)) + g(x^i)w(t) \\ z^i &= h_1^i(x^i); \quad y^i(t) = \beta^i h_2^i(x^i) \end{aligned} \qquad (38)$$

In (38), i=1,2 as the two sensors, the measurement data from each one are processed by an individual observer at each sensor node as is done in Section 3. The observer is given as

$$\begin{aligned} \dot{\hat{x}}^i(t) &= f^i(\hat{x}^i(t), \hat{x}^i(t-\tau)) + L^i(t)(y^i(t) - \hat{y}^i(t)) \\ \hat{z}^i &= h_1^i(\hat{x}^i); \; \hat{y}^i(t) = \beta^i h_2{}^i \hat{x}^i(t) \end{aligned} \qquad (39)$$

After the state estimates are obtained by each sensor node (of course concurrently by two processors), one can fuse these state estimates by using the formula as is done in the case of KF SVF

$$\hat{x}_f = \hat{x}^1 + \hat{P}^1\left(\hat{P}^1 + \hat{P}^2\right)^{-1}\left(\hat{x}^2 - \hat{x}^1\right) \qquad (40)$$

$$\hat{P}_f = \hat{P}^1 - \hat{P}^1\left(\hat{P}^1 + \hat{P}^2\right)^{-1}\hat{P}^{1^T} \qquad (41)$$

In (40) one has the individual state estimates obtained from the corresponding observer (39), that has processed the measurements from the corresponding sensor (i=1,2), and in (41), one has the Gramians (P for i=1,2), obtained by solving the corresponding matrix ORD equation (8). Hence, here, look upon P(.), i=1,2; as the weighting matrices used in the fusion rule (40). In the case of KF/EKF, P(.) are considered as the covariance matrices; however, for the SVF these are the appropriate weighting factors/matrices obtained from the covariance matrices, and happened to be the covariance matrices themselves. In the case of the nonlinear observers, one can consider these weighting factors/matrices as obtained from the Gramians (representing some uncertainty or dispersion of the estimate from the true value), and just happen to be the Gramians themselves, from the H infinity filtering theory. Here, also, it is ascertained that the theoretical development of Section 3 is equally applicable to the observers of (39), since each is an individual observer as in (2), and hence, by straightforward induction, the observer error dynamics of the systems with state delay, and randomly missing measurement data for the state vector fusion are locally asymptotically stable.

5. Simulation evaluation of the nonlinear observer

The presented HI based nonlinear observer is validated using numerical simulations carried out in MATLAB. The simulations are done for a period of 4 seconds with a sampling interval of 0.01 sec. Consider the following nonlinear dynamic system for simulation purpose

$$\begin{aligned}\dot{x}_1(t) &= -(x_1(t)+3.3)(x_1(t)+x_2(t)) \\ \dot{x}_2(t) &= -10x_2(t-\tau)+10x_2(t)-(3x_2(t-\tau)-10)x_1(t-\tau))\end{aligned} \qquad (42)$$

$y(t)=x_1(t)$

The nonlinear dynamic model in (42) is the prey, $x_1(t)$ – predator, $x_2(t)$ population dynamics model [11]. For, simulation as well as observer states, the dynamic equations (1), and (2) are solved by using Euler integration method, and hence, these equations and the Jacobians are appropriately represented in the discrete-form as

$$\begin{aligned}x_1(k) &= (-x_1(k-1)-3.3)(x_1(k-1)+x_2(k-1)) \\ x_2(k) &= -10x_2(k-2)+10x_2(k-1)-3x_2(k-2)x_1(k-2)+10x_1(k-2)\end{aligned} \qquad (43)$$

$$A_0(k)=\begin{bmatrix} -2x_1(k-1)-3.3-x_2(k-1) & -x_1(k-1)-3.3 \\ 0 & 10 \end{bmatrix} \qquad (44)$$

$$A_1(k)=\begin{bmatrix} 0 & 0 \\ -3x_2(k-2)+10 & -10-3x_1(k-2) \end{bmatrix} \qquad (45)$$

The state initial conditions for the simulation and the observer are chosen appropriately. To implement the observer algorithm, one needs to solve the matrix ORD equation (8), and for this use the following transformation method [1]

$$a = P(t)\,d \qquad (46)$$

Then using (46) in (8), obtain the

differential equations

$$\dot{d} = -A_0^T d + (b^2 H^T H - \frac{1}{\gamma^2} C^T C)\, a \qquad (47)$$

$$\dot{a} = (BB^T + A_1 A_1^T)\mathrm{d} + A_0 a \qquad (48)$$

The equations (47) and (48) are solved by using the (state-) transition matrix technique [1], for a and d, then using these in (46), obtain P(t). The performance of the observer with missing measurements at some level is illustrated in Figure 1. Figure 2 depicts the convergence of the eigenvalues of the matrix P (left graph), and true and predicted measurements. The performance of the observer with missing measurement data at some level and with the gamma factor used is illustrated in Figure 3. Figure 4 depicts the convergence of the eigenvalues of the matrix P (left graph), and the true and predicted measurements when the measurement data are missing at some level, and with gamma factor used. Table 1 shows the percentage fit errors for the states x_1, and x_2 and the measurement data fit error computed by using the formula: PFE=100*norm(error)/norm (true signal). From Figures 1-4, it is clear that the proposed nonlinear observer is asymptotically stable when some measurement data are missing at certain level. From Table 1 it can be seen that the PFEs of the HI based nonlinear observer are nearly similar or slightly more compared to the nonlinear observer when no gamma factor is used; in some cases even it is lesser as shown in bold numbers. The case of the no gamma factor corresponds to the nonlinear observer that is based on the usage of observer gain from the continuous time KF theory. Obviously, the measurements' PFEs show higher values when these data are missing. The studies presented in the present paper, and in refs. [11]-[13] pave the way for bringing synergy in the theory of observers, extended Kalman filter and H_∞ filtering.

6. Concluding remarks

In this paper, some new results on nonlinear observers for system with state delay and randomly missing measurements based on the H-infinity filter theory are presented.

Specifically, the asymptotic stability result has been presented for this HI based observer using the Lyapunov energy functional. The performance of the observer has been validated using prey-predator population dynamics model simulated in MATLAB. This exercise shows that despite some measurement data are missing, the performance of the proposed H infinity-based nonlinear observer is largely satisfactory and also corroborates the asymptotic results derived using the Lyapunov energy (LE) functional, as demonstrated and validated by the behaviour of the eigenvalues of the Gramian matrix. Also, the structures of the HI based nonlinear observers for the system with state delay and randomly missing data have been presented, for measurement level fusion and state vector fusion, in comparison with the conventional methods. Here, again for these observer based fusion schemes, the same asymptotic theoretical result holds true because of their non-complicated structures.

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Table 1 PFE metrics for the nonlinear observers

Data missing	No gamma factor used			Gamma factor used $\gamma > 0$ (HI based observer)		
	State x_1	State x_2	Meas. y	State x_1	State x_2	Meas. y
No	4.32	4.95	6.2	4.96	**4.6**	6.6
Yes, 5 %	4.2	4.76	23.4	4.94	**4.3**	23.61
Yes, 20 %	5.86	7.32	49.7	**5.5**	**5.6**	50.35

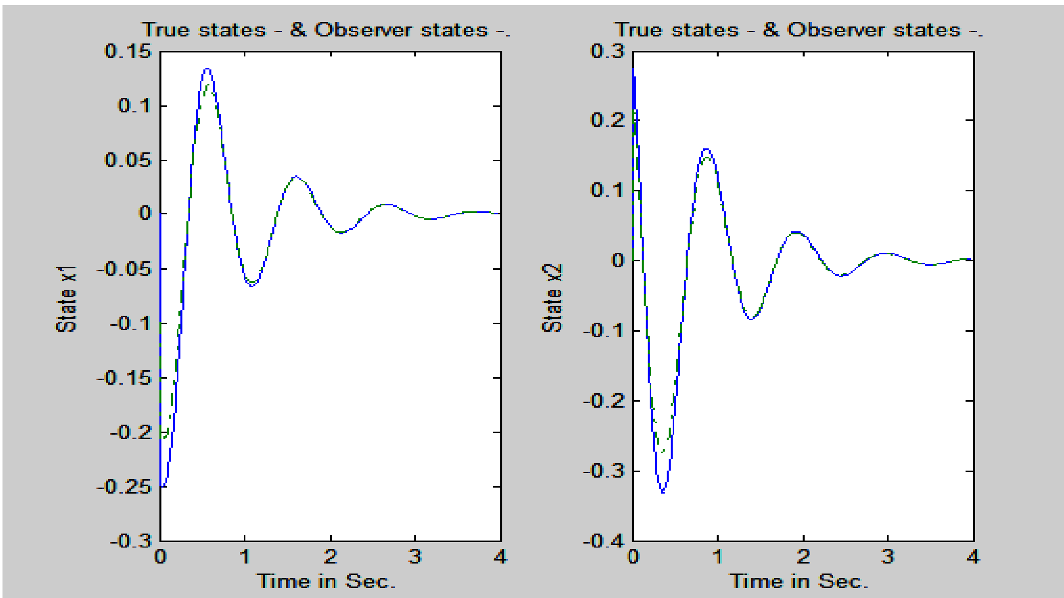

Figure 1. Time history match of the true (-) and observer states (-.); with missing data (level 20%, No gamma factor used).

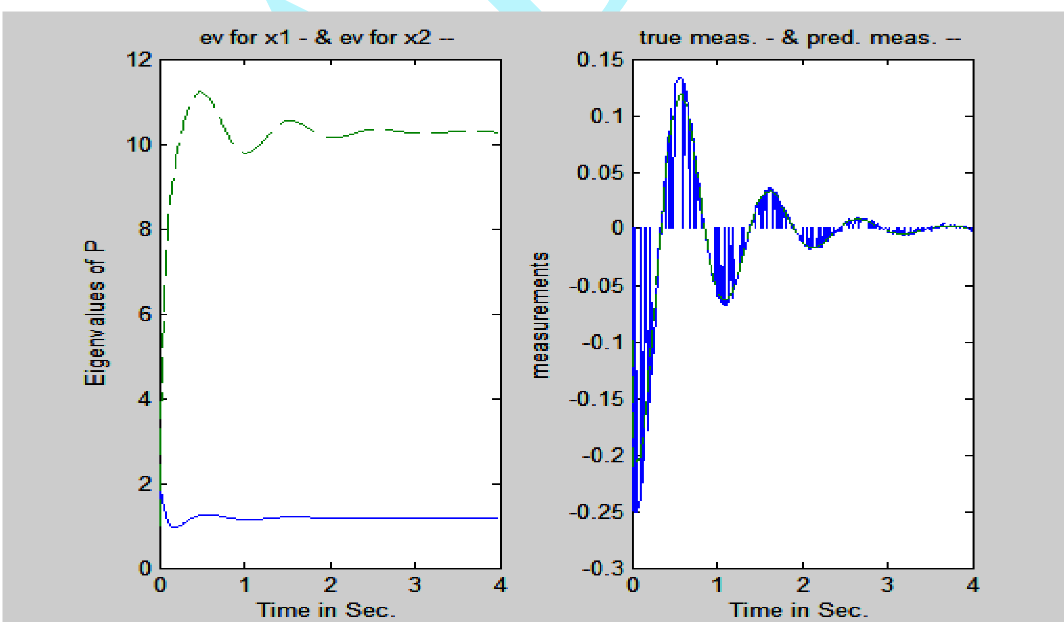

Figure 2. Eigenvalues of the P showing convergence/satisfaction of the condition of asymptotic

stability, related to state x_1 (-), & state x_2 (--); left graph; and the true (-), and predicted measurements (--); with missing data (level 20%, No gamma factor used).

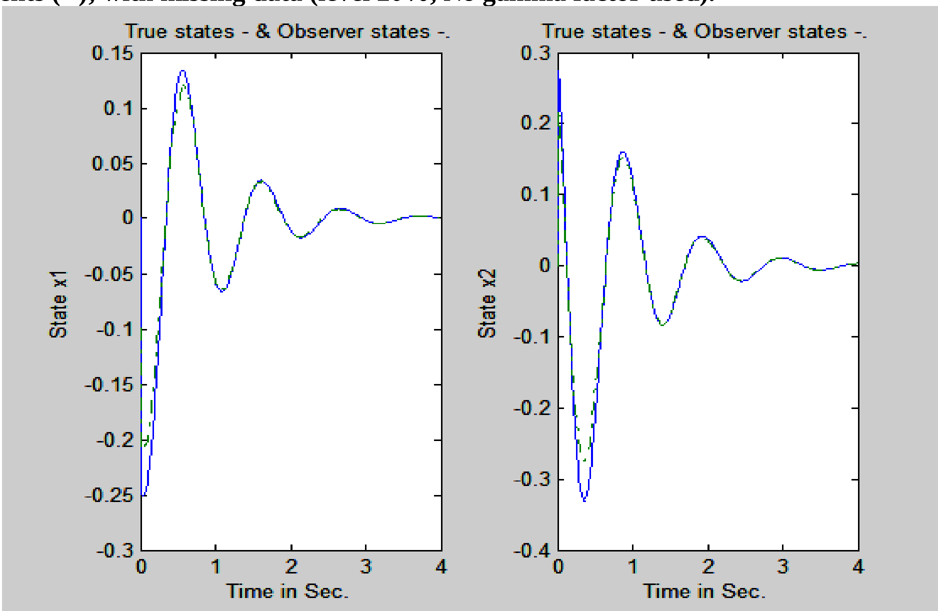

Figure 3. Time history match of the true (-) & observer (-.) states; with missing data (level 20%, with gamma factor used).

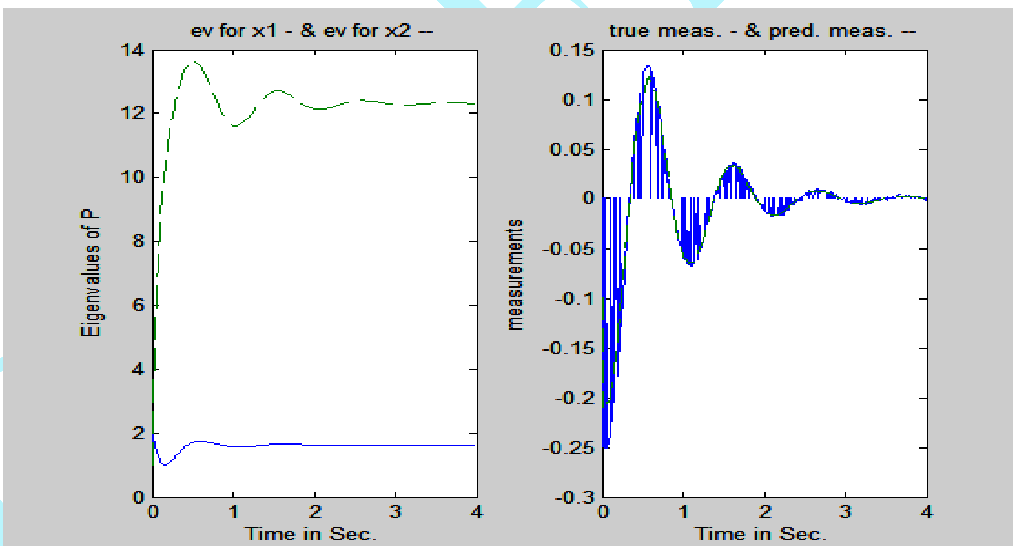

Figure 4. Eigenvalues of the P showing convergence/satisfaction of the condition of asymptotic stability, related to state x_1 (-), & state x_2 (--); left graph; and the true (-), and predicted measurements (--); with missing data (level 20%, with gamma factor used).