CBCS SCHEME

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BMATS201

Second Semester B.E./B.Tech. Degree Examination, June/July 2025

Mathematics – II for CSE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	**M**	**L**	**C**
Q.1	**a.**	Evaluate $$\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$$	**07**	**L3**	**CO1**
	b.	Evaluate $$\int_{0}^{\infty}\int_{x}^{\infty}\frac{e^{-y}}{y}\,dy\,dx$$ by changing the order of integration.	**07**	**L3**	**CO1**
	c.	Show that $$\beta(m,n)=\frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$$	**06**	**L2**	**CO1**
		OR			
Q.2	**a.**	Evaluate $$\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}}(x^2+y^2)\,dy\,dx$$ by changing to polar coordinates.	**07**	**L3**	**CO1**
	b.	Using double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	**07**	**L3**	**CO1**
	c.	Using mathematical tools, write the code to find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $6x + 3y + 2z = 6$.	**06**	**L3**	**CO5**
		Module – 2			
Q.3	**a.**	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2).	**07**	**L2**	**CO2**
	b.	Evaluate div $\vec{F}$ and curl $\vec{F}$ at the point (1, 2, 3), given $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$	**07**	**L3**	**CO2**
	c.	Express the vector $\vec{F} = 2x\,\hat{i} - 3y^2\,\hat{j} + zx\,\hat{k}$ in cylindrical form.	**06**	**L3**	**CO2**
		OR			
Q.4	**a.**	Find the directional derivative of $\phi(x, y, z) = x^2yz + yxz^2$ at (1, –2, 1) in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.	**07**	**L2**	**CO2**
	b.	Show that the vector $$\vec{F} = \frac{x\,\hat{i} + y\,\hat{j}}{x^2 + y^2}$$ is both solenoidal and irrotational.	**07**	**L3**	**CO2**
	c.	Using mathematical tool, write the code to find the gradient of $xy^3 + yz^3$.	**06**	**L3**	**CO5**

<table>
<tr><td colspan="6">Module – 3</td></tr>
<tr><td>Q.5</td><td>a.</td><td>Show that the set $W = \{(x, y, z) / x + y + 2z = 0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$.</td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Find the basis and dimension of the subspace spanned by the vectors $\{(1, -1, 0), (0, 3, 1), (1, 2, 1), (2, 4, 2)\}$ in R^3.</td><td>07</td><td>L3</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Find the matrix of the linear transformation $T : V_2(R) \rightarrow V_3(R)$ such that $T(-1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$.</td><td>06</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.6</td><td>a.</td><td>Determine whether the following set of vectors in 2×2 matrix space is linearly independent or linearly dependent:
$S = \{v_1, v_2, v_3,\}$ where $v_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$</td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Prove that $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (x + y, y + 2z)$ is a linear transformation.</td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Verify the Rank – nullity theorem for the linear transformation $T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (y - x , y - z)$.</td><td>06</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>Q.7</td><td>a.</td><td>Find a real root of the equation $x \log_{10} x = 1.2$ by regula falsi method corrected to 4 decimal places between (2.5, 3). Carry out 4 iterations.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>From the following table of half yearly premium for policies maturing at different ages, estimate the premium for the policies maturing at the age of 46.
<table>
<tr><td>Age :</td><td>45</td><td>50</td><td>55</td><td>60</td><td>65</td></tr>
<tr><td>Premium (in Rs.)</td><td>114.84</td><td>96.16</td><td>83.32</td><td>74.48</td><td>68.48</td></tr>
</table></td><td>07</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Use Trapezoidal rule to estimate the integration
$\int_0^1 \frac{1}{1+x^2} dx$ by taking 10 equal intervals.</td><td>06</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>Find by Newton's Raphson method, the root of the equation $\cos x = x e^x$ near to 0.5 , corrected to 4 decimal places.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Using Newton's divided difference interpolation, find the interpolating polynomial of the given data:
<table>
<tr><td>x :</td><td>- 4</td><td>- 1</td><td>0</td><td>2</td><td>5</td></tr>
<tr><td>f(x) :</td><td>1245</td><td>33</td><td>5</td><td>9</td><td>+1335</td></tr>
</table></td><td>07</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Evaluate using Simpson's $(1/3)^{rd}$ rule,
$\int_4^{5.2} \log x \, dx$ by dividing the range (4, 5.2) into seven ordinates.</td><td>06</td><td>L3</td><td>CO4</td></tr>
</table>

Module – 5

<table>
<tr><td>Q.9</td><td>a.</td><td>Solve $y' = 3x + y^2$; $y(0) = 1$ using Taylor's series method at $x = 0.1$ and $x = 0.2$.</td><td>07</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y = 1$ when $x = 0$. Find an approximate y at $x = 0.1$ using Euler's modified method. (Use modified formula twice).</td><td>07</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>From the data given below find y at $x = 1.4$ using Milne's method. Given $\frac{dy}{dx} = x^2 + \frac{y}{2}$.
<table><tr><td>x :</td><td>1</td><td>1.1</td><td>1.2</td><td>1.3</td></tr><tr><td>y :</td><td>2</td><td>2.2156</td><td>2.4549</td><td>2.7514</td></tr></table></td><td>06</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6" align="center">OR</td></tr>
<tr><td>Q.10</td><td>a.</td><td>Using Runge – Kutta method of order 4, find y at $x = 0.2$, given $\frac{dy}{dx} = \frac{x^2 + y^2}{10}$; $y(0) = 1$ taking $h = 0.1$.</td><td>07</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Using modified Euler's method solve $y' = 3x + \frac{y}{2}$ with $y(0) = 1$ taking $h = 0.2$ at $x = 0.2$. (Use modified Euler's formula twice).</td><td>07</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Using mathematical tools, write the code to solve $\frac{dy}{dx} = x^2y - 1$; $y(0) = 1$ by Taylor's series method at $x = 0.1\ (0.1)\ 0.3$</td><td>06</td><td>L3</td><td>CO5</td></tr>
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