CBCS SCHEME

USN

21MAT41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025

Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define analytic function and derive Cauchy–Riemann equations in polar form. **(06 Marks)**

b. If f(z) is analytic, show that :

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2.$$ **(07 Marks)**

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z+2)}dz$, where c is the circle $|z| = 3$. **(07 Marks)**

OR

2 a. Show that $f(z) = z + e^z$ is analytic, and hence find its derivative interms of z. **(06 Marks)**

b. Find the analytic function f(z) = u + iv, given that $u = x^2 - y^2 + \frac{x}{x^2+y^2}$ by Milne Thomson method. **(07 Marks)**

c. State and prove Cauchy's integral formula. **(07 Marks)**

Module-2

3 a. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. **(06 Marks)**

b. Obtain the series solution of Bessel's differential equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y = 0.$$ **(07 Marks)**

c. Express $x^3 - 5x^2 + x + 2$ in terms of Legendre polynomials. **(07 Marks)**

OR

4 a. Obtain the series solution of Legendre differential equation :

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0.$$ **(06 Marks)**

b. Express x^3+2x^2-4x+5 in terms of Legendre polynomial. **(07 Marks)**

c. Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$. **(07 Marks)**

Module-3

5 a. Find Karl Pearson's coefficient of correlation for the following data :

x	10	14	18	22	26	30
y	18	12	24	6	30	36

(06 Marks)

b. Fit a straight line y = ax + b for the data :

x	5	10	15	20	25
y	16	19	23	26	30

(07 Marks)

c. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(07 Marks)

OR

6 a. Ten students got the following percentage of marks in two subjects x and y. Compute their rank correlation coefficient.

Marks in x	78	36	98	25	75	82	90	62	65	39
Marks in y	84	51	91	60	68	62	86	58	53	47

(06 Marks)

b. Compute the means $\bar{x}, \bar{y}$ and the correlation coefficient r from the given regression lines,
4x – 5y + 33 = 0
20x – 9y = 107. **(07 Marks)**

c. Fit a Parabola $y=ax^2+bx+c$ by the method of least squares for the following data :

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

(07 Marks)

Module-4

7 a. A random variable X has the following probability function :

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

Find K.
Also find :
i) $P(x \geq 5)$
ii) $P(3 < x \leq 6)$. **(06 Marks)**

b. Derive the mean and variance of Poisson distribution. **(07 Marks)**

c. The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that :
i) Exactly two are defective
ii) Atleast two are defective
iii) None of them are defective. **(07 Marks)**

OR

8 a. A random variable X has the density function :

$$f(x) = \begin{cases} kx^2 & -3 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Evaluate K.
Also find :
i) $P(1 \leq x \leq 2)$
ii) $P(x \leq 2)$
iii) $P(x > 1)$. **(06 Marks)**

b. The probability that an individual suffers a bad reaction from as injection is 0.001. Find the probability that out of 2000 individuals.
i) Exactly three
ii) More than 2 will get bad reaction. **(07 Marks)**

c. In a normal distribution 31% of the items are under 45 and 8% over 64. Find the mean and S.D. Given A(0.5) = 0.19 and A(1.4) = 0.42. **(07 Marks)**

Module-5

9 a. The joint probability distributions of two random variables are given below :

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Determine :
i) Marginal distributions of x and y
ii) E[X] and E[Y]
iii) Verify X and Y are independent. **(06 Marks)**

b. Define :
i) Null hypothesis
ii) Type – I and Type – II errors
iii) Level of significance. **(07 Marks)**

c. A certain stimules administered to each of 12 patients resulted in the following changes in the blood pressure, 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that stimules will increase the blood pressure [$t_{0.05}$ for 11 d.f is 2.201]. **(07 Marks)**

OR

10 a. The joint probability distribution of two random variables X and Y are given below :

X \ Y	-3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine :
i) E(X) and E(Y)
ii) E[X Y]
iii) COV(X, Y). **(06 Marks)**

b. Find the student 't' test for the following variable values in a sample of 8 are – 4, –2, –2, 0, 2, 2, 3, 3. Taking the mean of the universe to be zero. **(07 Marks)**

c. The theory predicts the proportion of beans in the four group A, B, C and D should be 9 : 3 : 3 : 1. In an experiment among 1600 beans, the number in the four groups were 882, 313, 287 and 118. The goodness of fit χ^2 values of above data is approximately equal to? $\left(\chi^2_{0.05} = 5.99\right)$. **(07 Marks)**

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