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18MAT41

Fourth Semester B.E. Degree Examination, June/July 2025
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

***Note:* Answer any FIVE full questions, choosing ONE full question from each module.**

Module-1

1 a. Derive Cauchy-Riemann equations in the polar form. **(06 Marks)**

b. Show that $W = f(z) = \log z\ (z \neq 0)$ is analytic and hence find $\frac{d\omega}{dz}$. **(07 Marks)**

c. Find the analytic function $f(z) = u + iv$, whose imaginary part is $V = \left(r - \frac{1}{r}\right)\sin\theta$. **(07 Marks)**

OR

2 a. If $W = f(z) = \phi(x, y) + i\psi(x, y)$ represents the complex potential of an electrostatic field, when $\psi = (x^2 - y^2) + \frac{x}{x^2 + y^2}$. Find f(z) and hence determine ϕ. **(06 Marks)**

b. If $f(z) = u(x, y) + i\,v(x, y)$ is an analytic function, show that the family of curves $u(x, y) = C_1$ and $V(x, y) = C_2$ intersect each other orthogonally. **(07 Marks)**

c. If f(z) is a regular function of z, show that $\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$ **(07 Marks)**

Module-2

3 a. Show that the transformation $W = f(z) = e^z$ maps straight lines parallel to the co-ordinate axes in the Z-plane onto orthogonal trajectories in the W-plane and sketch the region. **(06 Marks)**

b. Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $W = 0, i, \infty$. **(07 Marks)**

c. State and prove Cauchy's Integral Formula. **(07 Marks)**

OR

4 a. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 3$. **(06 Marks)**

b. Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along the real axis upto 2 and then vertically to 2 + i. **(07 Marks)**

c. Find the bilinear transformation which maps the points $z_1 = -1$, $z_2 = 0$, $z_3 = 1$ into the points $W_1 = 0$, $W_2 = i$, $W_3 = 3i$ **(07 Marks)**

Module-3

5 a. The probability density function of variate X is given by the following table :

x	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

For what value of K, this represents a valid probability distribution? Also find $P(x \geq 5)$ and $P(3 < x \leq 6)$. **(06 Marks)**

b. The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that, (i) Exactly 2 are defective (ii) atleast 2 are defective (iii) None of them are defective **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be,

(i) Less than 65.
(ii) More than 75.
(iii) 65 to 75. **(07 Marks)**

OR

6 a. The probability density function of a random variable X is $f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$.

Find : (i) The value of K (ii) $P(x \leq 1)$ (iii) $P(x > 1)$
(iv) $P(1 < x < 2)$ (v) Mean (vi) Variance **(06 Marks)**

b. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction. **(07 Marks)**

c. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for,

(i) 10 minutes or more
(ii) Less than 10 minutes
(iii) Between 10 and 12 minutes. **(07 Marks)**

Module-4

7 a. Ten competitors in a beauty contest are ranked by two judges in the following order. Compute the coefficient of rank correlation. **(06 Marks)**

Judge A	1	6	5	3	10	2	4	9	7	8
Judge B	6	4	9	8	1	2	3	10	5	7

b. Fit a parabola $y = a + bx + cx^2$ for the following data :

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(07 Marks)

c. If θ is the acute angle between the lines of regression, then show that $\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1 - r^2}{r}\right)$. **(07 Marks)**

OR

8 a. With usual notation, compute $\bar{x}$, $\bar{y}$ and r from the following equation of the regression lines: $2x+3y+1=0$, $x+6y-4=0$. **(06 Marks)**

b. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(07 Marks)

c. Fit a best fitting curve in the form $y = ax^b$ for the following data :

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(07 Marks)

Module-5

9 a. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure. 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05}$ for 11 d.f = 2.201) **(06 Marks)**

b. The joint distribution of two random variables X and Y is as follows :

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Compute the following :

(i) E(X) and E(Y)

(ii) E(XY)

(iii) σ_x and σ_y

(iv) Cov(x, y)

(v) $\rho(x, y)$ **(07 Marks)**

c. A random sample for 1000 workers in company has mean wage of Rs.50 per day and S.D of Rs.15. Another sample of 1500 workers from another company has mean wage of Rs.45 per day and S.D. of Rs.20. Does the mean rate of wages varies between the two companies? Find the 95% confidence limits for the difference of the mean wages of the population of the two companies. **(07 Marks)**

OR

10 a. Explain the terms :

(i) Null Hypothesis

(ii) Confidence intervals

(iii) Type – I and Type – II errors **(06 Marks)**

b. Fit a Poisson distribution for the following data and test the goodness of fit given that $\chi^2_{0.05} = 7.815$ for 3 d.f. **(07 Marks)**

x	0	1	2	3	4
f	122	60	15	2	1

c. A 'die' is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the die cannot be regarded as an unbiased one. **(07 Marks)**

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