CBCS SCHEME

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21MATME41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025
Complex Analysis, Probability and Linear Programming

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Statistical tables permitted.

Module-1

1 a. Show that the function $f(z)=|z|^2$ is differentiable but not analytic at the origin. **(06 Marks)**

b. Find the analytic function f(z) = u + iv. Given $u-v=\dfrac{\cos x+\sin x-e^{-y}}{2(\cos x-\cosh y)}$ and $f\left(\dfrac{\pi}{2}\right)=0$. **(07 Marks)**

c. If f(z) is analytic function of Z, prove that $\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$. **(07 Marks)**

OR

2 a. With usual notations, derive the Cauchy-Riemann equations in polar form. **(06 Marks)**

b. Show that W = logz is analytic every were except at z = 0 and hence find its derivative. **(07 Marks)**

c. Find an analytic function f(z) = u + iv whose imaginary part is $v = r^2\cos2\theta - r\cos\theta + 2$. **(07 Marks)**

Module-2

3 a. Discuss the transformation $w=z+\dfrac{1}{z}$. **(06 Marks)**

b. Evaluate the integral $\int_c |z|^2 dz$, were c is the square having vertices at the origin 0 and the points P(1, 0), Q(1, 1) and R(0, 1). **(07 Marks)**

c. Find the bilinear transformation that transforms the points $z_1 = i$, $z_2 = 1$, $z_3 = -1$ onto the points $w_1 = 1$, $w_2 = 0$, $w_3 = \infty$ respectively. **(07 Marks)**

OR

4 a. If c is the simple closed curve in the complex plane, evaluate the following :

i) $\int_c \dfrac{dz}{z-a}$ ii) $\int_c \dfrac{dz}{(z-a)n}$, n = 2,3,..... on the cases a is outside c and a is inside c. **(06 Marks)**

b. Using the Cauchy's integral formula evaluate $\int_c \dfrac{z}{z^2-3z+2}dz$ were c is the circle

i) $|z-2|=\dfrac{1}{2}$ and ii) $|z|=1$. **(07 Marks)**

c. Discuss the transformation $w = z^2$. **(07 Marks)**

Module-3

5 a. The probability distributive function P(X) of a variate x is given by the following table :

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

For what value of K, does this represent a valid probability distribution? Find $P(x < 4)$, $P(x \geq 5)$ and $P(3 < x \leq 6)$. **(06 Marks)**

b. The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men, now aged 60,
i) Exactly 9 will live to be 70
ii) At most 9 will live to be 70
iii) At least 7 will live to be 70? **(07 Marks)**

c. If x is an exponential variate with mean 4, evaluate the following :
i) $P(0 < x < 1)$ ii) $P(x > 2)$ iii) $P(-\infty < x < 10)$. **(07 Marks)**

OR

6 a. A random variable x has the density function $P(x) = \begin{cases} kx^2, & 0 \leq x \leq 3 \\ 0, & \text{elsewere} \end{cases}$, evaluate k and find
i) $P(x \leq 1)$ ii) $P(1 \leq x \leq 2)$ iii) $P(x > 1)$ **(06 Marks)**

b. A certain screw making machine has a chance of producing 2 defective out of 1000. The screws are packed in boxes of 100. Using poison distribution, find the approximate number of boxes containing i) no defective screw ii) two defective screws in a consignment of 5000 boxes. **(07 Marks)**

c. In a normal distribution 7% are under 35 and 89% are over 63. Find the mean and standard deviation, given that $A(1.23) = 0.39$ and $A(1.48) = 0.43$, in the usual notation. **(07 Marks)**

Module-4

7 a. Use the simplex method to maximize $z = 3x + 4y$ subject to the constraints :
$2x + y \leq 40$, $2x + 5y \leq 180$, $x \geq 0$, $y \geq 0$. **(10 Marks)**

b. Use Big-M method solve the L.P.P. minimize $z = 2x + 3y$ subject to the constraints $x + y \geq 5$, $x + 2y \geq 6$, $x \geq 0$, $y \geq 0$. **(10 Marks)**

OR

8 a. Define the following terms: A linear programming problem, basic solution, basic feasible solution, optimal solution, artificial variables of an LPP. **(10 Marks)**

b. Solve the following LP problem by the using two-phase simplex method.
Minimize $z = x - 2y - 3z$
Subject to the constraints $-2x + y + 3z = 2$,
$2x + 3y + 4z = 1$ and
$x, y, z \geq 0$ **(10 Marks)**

Module-5

9 a. Determine an initial basic feasible solution to the following transportation problem by using i) Least cost method ii) North west corner method.

	D_1	D_2	D_3	D_4	Supply
S_1	21	16	15	3	11
S_2	17	18	14	23	13
S_3	32	27	18	41	19
	6	10	12	15	

(10 Marks)

b. ABC company is engaged in manufacturing 5 brands of packed snacks. It is having five manufacturing setups, each capable of manufacturing any of its brands, one at a time. The cost make a brand on these setups vary according to the following table:

	S_1	S_2	S_3	S_4	S_5
B_1	4	6	7	5	11
B_2	7	3	6	9	5
B_3	8	5	4	6	9
B_4	9	12	7	11	10
B_5	7	5	9	8	11

Find assignment of brands to snacks are optimum. **(10 Marks)**

OR

10 a. Determine an initial basic feasible solution of the following transportation problem by using Vogel's approximation method:

	D_1	D_2	D_3	D_4	Supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

(10 Marks)

b. Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is known and given in the following table:

Job

	Men \ Job	I	II	III	IV	V
	A	2	9	2	7	1
	B	6	8	7	6	1
Men	C	4	6	5	3	1
	D	4	2	7	3	1
	E	5	3	9	5	1

Find assignment of men to jobs that will minimize the total time taken. **(10 Marks)**

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