CBCS SCHEME

USN | | | | | | | | | | 18EC44



# Fourth Semester B.E. Degree Examination, June/July 2025
# Engineering Statistics and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***

## Module-1

1 a. The pdf for the random variable X is given by

$$f_X(x) = \begin{cases} 0.5303\sqrt{x}, & 0 < x < 2 \\ 0, & \text{Otherwise} \end{cases}$$

Find:
i) The mean
ii) The mean of the square
iii) The variance of the random variable X. **(06 Marks)**

b. The following is the Pdf for the random variable U

$$f_U(u) = \begin{cases} c\exp\left(\frac{-u}{2}\right), & 0 \le u < 1 \\ 0, & \text{Otherwise} \end{cases}$$

Find the value of C and evaluate $F_U(0.5)$. **(06 Marks)**

c. Define a binomial random variable. Obtain the characteristic function of a binomial distribution and hence find mean and variance using the characteristic function. **(08 Marks)**

**OR**

2 a. A random variable X has a poisson distribution with a mean of 3, find $P(1 \le x \le 3)$. **(06 Marks)**

b. The probability distribution of a discrete random variable is as shown below:

| K | -0.25 | 0 | 1 | 2 | 3.75 |
|---|---|---|---|---|---|
| P(X = K) | 0.2 | C | 0.4 | 0.1 | 2C |

Find the value of C and P $\{(X > 1) / (X \ge 0)\}$. **(06 Marks)**

c. Derive mean, variance and characteristic function for exponential distributed random variable. **(08 Marks)**

## Module-2

3 a. The joint Pdf $f_{XY}(x, y) = c$, a constant, when $(0 < x < 3)$ and $(0 < y < 3)$ and is 0 otherwise.
i) What is the value of the constant c?
ii) What are the Pdfs for X and Y?
iii) What is $F_{XY}(x, \infty)$ when $(0 < x < 3)$ and $(0 < y < 3)$? **(06 Marks)**

b. The random variable X is uniformly distributed between ±1.

i) Find the mean and variance of Y if $Y = \frac{1}{37}\sum_{i=1}^{37} X_i$

ii) Find the mean and variance of Z if $Z = \sum_{i=1}^{37} X_i$

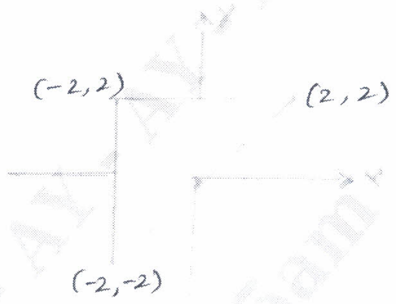
In these two sums, the $x_i$'s are IID. **(06 Marks)**

c. X and Y are correlated random variables with a correlation coefficient of $\rho = 0.6$, $\mu_X = 3$, $Var[X] = 49$, $\mu_Y = 144$, $Var[Y] = 144$. The random variables U and V are obtained using $U = X + CY$ and $V = X - CY$. What values can C have if U and V are uncorrelated? **(08 Marks)**

**OR**

4 a. Prove that sum of the two independent Gaussian random variables is also Gaussian. **(06 Marks)**

b. Shown in Fig.Q.4(b) is a region in the x, y plane where bivariate Pdf $f_{xy}(x, y) = c$, elsewhere the Pdf is 0.




i) Find the value of c

ii) Evaluate $F_{xy}(1, 1)$

Fig.Q.4(b) **(06 Marks)**

c. Write a note on:

i) Students t random variable

ii) Chi-square random variable. **(08 Marks)**

## Module-3

5 a. For the random process $X(t) = A\cos(w_c t + \theta)$, A and $W_c$ are constants $\theta$ is a random variable, uniformly distributed between $\pm\pi$. Show that this process is ergodic. **(06 Marks)**

b. Define random process and discuss the terms Strict-Sense Stationary (SSS) and Wide Sense Stationary WSS associated with a random process. **(06 Marks)**

c. The random process X(t) has the auto correlation function

$$R_x(t) = \begin{cases} 10\left(1 - \dfrac{|\tau|}{\tau_N}\right), & -\tau_N \le \tau \le \tau_N \\ 0, & \text{Otherwise} \end{cases}$$

The random process Y(t) is independent of X(t) and has the auto correlation function $R_Y(\tau) = \frac{13\sin(W_B\tau)}{W_B\tau}$, where $\frac{2\pi}{W_B} \ge \tau_N$. The random process $Z(t) = X(t) + Y(t)$. For Z(t), find its auto correlation function, its total power, its dc power. **(08 Marks)**

**OR**

6 a. x(t) and y(t) are independent, jointly wide-sense stationary random process given by $x(t) = A\cos(w_1 t + \theta_1)$ and $y(t) = B\cos(w_2 t + \theta_2)$. If W(t) = X(t) Y(t) then find ACF $R_w(\tau)$. **(06 Marks)**

b. The random process X(t) and Y(t) are jointly wide-sense stationary and they are independent. Given that W(t) = X(t) Y(t) and $R_X(\tau) = 10\exp\left(\frac{-|\tau|}{3}\right)$, $-\infty < \tau < \infty$,

$$R_y(\tau) = \begin{cases} 11\left(\dfrac{3-|\tau|}{3}\right), & -3 \le \tau < 3 \\ 0, & \text{Otherwise} \end{cases}$$

For W(t), find its autocorrelation function, its total power, its dc power and its ac power. **(06 Marks)**

c. Suppose that the PSD input to a linear system is $S_X(w) = k$. The cross-correlation of the input X(t) with the output Y(t) of the linear system is found to be

$$R_{XY}(\tau) = K\begin{cases} e^{-\tau} + 3e^{-2\tau}, & \tau \ge 0 \\ 0, & \tau < 0 \end{cases}$$

What is the power filter function $|H(3w)|^2$? **(08 Marks)**

## Module-4

7 a. Determine whether the vectors (1, 1, 1), (2, 2, 0) and (3, 0, 0) is linearly independent or dependent. **(06 Marks)**

b. Find the basis and dimension of the subspace $V_3(R)$ spanned by the vectors (1, -2, 3), (1, -3, 4) and (-1, 1, -2). **(06 Marks)**

c. If $u_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ; $u_2 = \begin{pmatrix} -1/4 \\ 1 \\ 1/4 \end{pmatrix}$ ; $u_3 = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$

Show that $u_1$, $u_2$, $u_3$ are pairwise orthogonal vectors. Hence find the orthonormal vectors of $u_1$, $u_2$, $u_3$. **(08 Marks)**

**OR**

8 a. Find the column space and null space of

$$A = \begin{bmatrix} 1 & -5 & 7 & 0 \\ 1 & -1 & 0 & 0 \\ -2 & 10 & -14 & 0 \end{bmatrix}$$

**(06 Marks)**

b. Find a least squares solution of the inconsistent system AX = b for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ ; $b = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$. **(06 Marks)**

c. Let W = span $\{X_1, X_2\}$ where

$X_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$, $X_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Construct an orthogonal basis $[v_1, v_2]$ for w. **(08 Marks)**

## Module-5

9 a. Reduce the matrix A to upper triangular matrix U and find the determinant by product of pivots,

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$$

(05 Marks)

b. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ (05 Marks)

c. Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (10 Marks)

**OR**

10 a. Write the properties of determinants. (05 Marks)

b. Test the matrix for positive definiteness of

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(05 Marks)

c. Find a singular value decomposition of $A = \begin{pmatrix} 2 & -1 \\ 2 & 2 \end{pmatrix}$. (10 Marks)

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